Theoretical Courses

on

Aeronautics for Airship Pilots

(From the French)

Translated by the Air Service, A. E. F.
July, 1919

WASHINGTON
GOVERNMENT PRINTING OFFICE
1920

War Department

Document No. 909

Office of The Adjutant General

WAR DEPARTMENT,
WASHINGTON, *July 8, 1919.*

The following "Theoretical Courses on Aeronautics for Airship Pilots" (from the French), translated by the Air Service, A. E. F., United States Army, is approved and published for the information and guidance of all concerned.

[062.1, A. G. O.]

BY ORDER OF THE SECRETARY OF WAR:

PEYTON C. MARCH,
General, Chief of Staff.

OFFICIAL:

P. C. HARRIS,
The Adjutant General.

Digitized by the Internet Archive
in 2015

https://archive.org/details/theoreticalcours00unse

[Translation No. 3222.]

THEORETICAL COURSES ON AERONAUTICS.

Compiled by Lieut. MENEZ and revised by Capt. JOUX (1917).

THE PRINCIPLE OF ARCHIMEDES.

The science of ballooning deals with machines lighter than air—that is, the total weight of such machines is less than the gravity of the volume of air which they displace. They therefore rise and are sustained by virtue of the principle of Archimedes, the enunciation of which is well known. Any body entirely immersed in a fluid, in equilibrium under the sole action of its weight, would be subjected to a thrust, on the part of that fluid, in a vertical upward direction equal to the weight of the fluid displaced and applied to the center of gravity of the fluid mass.

We shall extend the principle to the case of fluids that are not in a state of equilibrium and apply its statement to gaseous fluids.

The principle of Archimedes.—The resultant of the pressures exerted on a solid immersed in gas is an upward thrust equal to the volume of gas displaced and applied to the center of gravity of that volume. It is called the "center of thrust" or "stream-linecenter."-

The quantitative valuations to which the application of the principle gives rise consequently include the expression of weight of a certain gaseous volume. With a view to such application, we shall briefly recall the physical properties of gases.

ENUMERATION OF THE PHYSICAL PROPERTIES OF GASES.

Specific gravity.—The weight of the unit of volume of a body is generally called the "specific gravity" of that body. It is thus a force—namely, the effect of the weight on a closely determined fraction of the body.

In the C. G. S. system of units, the unit of volume being the cubic centimeter and the unit of force the dyne, the specific gravity is expressed in dynes per cubic centimeter. We have, for instance, the following values in Paris:

For water, 981 dynes per cubic centimeter.
For air, 1.269 dynes per cubic centimeter, air at 0° and at 760 mm.

In the kilogrammeter system of units, the unit of volume being the cubic meter and the unit of force the kilogram, the specific gravity is expressed in kilograms per cubic meter. For water and air we have, respectively, the quantities 1,000 kilograms per cubic meter and 1.293 cubic meters.

In dealing with gas the latter system of units prevails. The quantity of 1.293 kilos is constantly found in all calculations in which the gravity of gas is taken into account.

DENSITY.

The term of comparison of the ratio of a certain weight of the body B to the weight of the same volume of the body A is generally called the density of a body B in relation to a body A. Their ratio is an independent quantity of the system of units employed.

The standard body of comparison A being selected, the density of the body B is characteristic of the former; it is a constant quality of that body. The specific gravity is known by means of it. If a be the specific gravity of the body A, known once for all, and b the specific gravity of B, the density d of B in ratio to A is equivalent, by reason of the same definition, to $\frac{b}{a}$.

Inversely, the equality $b=a\ d$ gives the specific gravity b.

In the case of solids and of liquids, the standard body of comparison is water. To be precise, it must be pure water at 4° Centigrade and at a pressure of 760 mm.

Density of gases.—In the case of gases, the conditions of pressure and of temperature are of such importance that it is impossible for them not to be taken into account. They are inserted in the definition. The gas used for purposes of comparison is air under established conditions of purity. Practically, the composition of atmospheric air is remarkably constant, excepting for the proportion of steam it contains. This variability is eliminated by taking dry air as a term of comparison.

The density of a gas (it is understood to be in ratio to the air) is the definition given to the ratio between the gravity of a certain volume of that gas and the gravity of the same volume of dry air taken under the same conditions of temperature and pressure.

The ratio thus obtained is almost constant, whatever may be the conditions of temperature and pressure. When it is said to be constant, it is implied that the laws of compressibility are the same for all gases. This will be taken for granted.

The following are the constant qualities for some gases:

Chlorine	2.45
Carbonic gas	1.53
Oxygen	1.105

heavier than air.

Azote	.97
Acetylene	.91
Steam	.625(5/8)
Ammonia	.60
Methane	.56
Helium	.139
Hydrogen	.069

lighter than air.

Illuminating gas is an industrial mixture of a composition closely averaging that stated in the following list:

Gas	H	CH^4	CO	C^2H^4	Az	CO^2
Per cent in volume	45 to 52	32 to 40	4 to 6	4 to 5	3 to 5	1 to 2

The density is approximately 0.40.

The relation between the specific gravity and the density, noted in a general sense, holds good in the case of gas. We have thus the following numbers for specific weights under normal conditions of pressure and of temperature:

Hydrogen, b=1.293 kg.$\times$.069=.089 cubic meter.
Lighting gas, b=1.293 kg.$\times$.40 =.52 cubic meter (about).

NOTE.—The hydrogen produced in large quantities, as in the case of the inflation of balloons, can not be obtained chemically pure in utilization devices. Its specific gravity will therefore be higher than the preceding. One hundred grammes is practically the minimum for aerostatic hydrogen, taken at the temperature of 0° and under the pressure of 760 mm.

We shall now see the effect of variations of pressure on the specific gravity of a gas, commencing by varying the pressure only.

THE MARIOTTE LAW.

Compressibility of gases.—At a constant temperature, the volumes occupied by a gaseous mass are in inverse ratio to the pressures they sustain.

The mass, and consequently the total weight, being invariable, the specific gravity or gravity per unit of volume is evidently in inverse ratio to the volume.

The specific gravity and the pressure, both inversely proportional to the volume, are together directly proportionate.

These terms may be expressed by the following relations:

$$v.p. = \text{constant} = c \text{ (Mariotte).} \quad (1)$$

$$v.a. = \text{constant} = c' \text{ (total weight).} \quad (2)$$

By dividing the parts separately, we get:

$$\frac{a}{p}=\frac{c'}{c}\text{constant.}$$

If two different conditions of the gaseous mass be considered, characterized respectively by the elements

$$v_1,\ p_1,\ a_1,$$
$$v_2,\ p_2,\ a_2,$$

we get

$$\frac{v_1}{v_2}=\frac{p_2}{p_1}. \qquad (1')$$

$$\frac{v_1}{v_2}=\frac{a_2}{a_1}. \qquad (2')$$

Whence

$$\frac{a_2}{a_1}=\frac{p_2}{p_1},$$

or finally

$$a_2=a_1\times\frac{p_2}{p_1}. \qquad (3)$$

Let us take the particular case of air supposed to be maintained at 0°. Under a pressure of 760 mm. its specific gravity is known to be 1.293 kilometers. Under pressure of p millimeters of mercury, it will be equal to

$$a=1.293\times\frac{p}{760}.$$

THE LAW OF GAY-LUSSAC.

Calorific expansion of gases.—The effect of a rise of temperature on gases is to make them expand, like some other bodies. By reason of the compressibility of gases, to which we have just called attention, the variation of volume thus obtained can not be stated without knowing exactly what pressure is sustained. We shall begin by supposing such pressure to be kept constant.

The law of variation is the universal law; so long as the temperature varies within sufficiently narrow limits, the variation of volume is proportional to the initial volume and to the variation of temperature, as the elongation of a bar is proportional to its initial length and to the increase of temperature.

It may be stated that

$$v'-v=k.v\ (t'-t). \qquad (4)$$

If the particular case of a variation of temperature from 0° up to t° be considered, if v_6 and v are the two volumes of the gaseous mass at 0° and at t°, the formula will be as follows:

$$v-v_0=v_0t, \qquad (5)$$

whence $v = v_0 (1\ t)$ in expressing the value of the coefficient of proportionality, in this case, by α; α is called the "coefficient of expansion" of gas.

The substance of the law of Gay-Lussac is as follows: The coefficient of expansion is the same for all gases.

This law is very approximately correct in the case of gases which liquify at a very low temperature (among which hydrogen, azote, and oxygen may be particularly noted) and which are sometimes called perfect gases. It is sufficiently exact for other gases and steam when taken, as in the present case, with slight variations of temperature.

The value adopted for the coefficient α is $1/273 = .00366$.

If the temperature rises from 0° to t°, the volume thus increases by $t/273$ of its value.

Suppose a first value $t = 10$; the volume has increased by 10/273, that is, by about 1/27 or 36/1000 of its value.

Suppose that a second value $t' = 27$. From 0° to 27° the volume has increased by 27/273, that is, by about one-tenth of its value.

Let us prove that the coefficient of proportionality, which has been designated by two distinct letters in formulas (4) and (5), varies for this reason from one case to the other. We shall therefore express the two volumes v and v' at temperatures t and t' according to formula (5):

$$v = v_0\ (1\ t),$$
$$v' = v_0\ (1\ t').$$

Whence

$$v' - v = v_0\ (t' - t) = v_0 \frac{t' - t}{273}. \tag{6}$$

In the second part, we shall replace v_0 in function of v according to formula (5); there is still

$$v_0 = v \frac{1}{1t} = v \frac{273}{273t}.$$

This gives (6′)

$$v' - v = v_0 \frac{t' - t}{273} = v \frac{t' - t}{273 + t}.$$

A rise of temperature of 1° at constant pressure, starting from t°, therefore increases the volume by 1/273 of its value v_0 to 0° or from $1/273\ t$ of its value v at t°.

Thus $$K = \frac{1}{273 + t}.$$

For instance, when t is about 25 or 30°, K is about 1/300. It frequently happens, nevertheless, that a rise of temperature of 1° is said to make the volume increase by 1/273 of its momentary value, without its actual temperature being taken into account. This is an approximate statement to which there is, as we shall presently see, a correspondingly approximate calculation.

We shall now make the temperature and the pressure vary, successively to commence with, so that the laws of Mariotte and of Gay-Lussac may be applied in the same order:

1. The temperature alone varies from 0 to t : $v = v_0\ (1 + \alpha\ t)$.

2. Let the pressure alone be varied in order to bring the volume up to its value v_0. The condition thus obtained would be the result of heating from 0° to t° at constant volume. But the pressure has become p in such wise that

$$pv_0 = p_0 v = p_0 v_0\ (1 + \alpha\ t).$$

Thus

$$p = p_0\ (1 + \alpha\ t). \tag{7}$$

The law of variation of pressure with constant volume is the same as the law of expansion at constant pressure.

3. Let the volume and pressure be varied in any manner whatever at the same temperature t. According to the law of Mariotte, if P and V are the two actual values of pressure and volume, the result is necessarily

$$P.V = p_0 v_0 (1 + \alpha t). \tag{8}$$

This formula characterizes the most general transformation that any one gaseous mass can undergo. Two of the elements, volume, pressure and temperature, may be varied in an arbitrary manner, and the third results therefrom. If the pressure and temperature, in particular, be arbitrarily varied, the volume is given by the following formula:

$$V = v_0 \frac{p_0}{p} (1 + \alpha t).$$

The law of variation of specific gravity, general in this case, is directly deduced from this formula. We know, as a matter of fact, that the specific gravity varies in inverse proportion to the volume by reason of the constancy of the total weight. Therefore, if a_0 be the specific gravity at 0° under p_0 pressure, the specific gravity a, under pressure P and at a temperature t°, will be expressed as follows:

$$a = a_0 \frac{P}{P_0} \frac{1}{1 + \alpha t}. \tag{9}$$

This shows that we might already have deduced from formula (5) that, in the particular case when the pressure remains constant, the specific gravity varies in inverse proportion to the binomial of expansion.

Under any new conditions whatever of P′ and t', the specific gravity is thus expressed:

$$a' = a_0 \frac{P'}{p_0} \frac{1}{1 + \alpha t'}.$$

From this we deduce:

$$a' = a\frac{P'}{P}\,\frac{1+\alpha t}{1+\alpha t'}, \tag{10}$$

which relation gives the variation of the specific gravity when the temperature and the pressure pass from any two values to any two new values.

Artifice in calculation.—It must be recollected that if ϵ be a quantity that is small in ratio to 1, $\frac{1}{1+\epsilon}$ may be replaced by $1-\epsilon$, an approximation which amounts to neglect of the term of ϵ^2 in ratio to 1. We may thus replace $\frac{1}{1+\alpha t}$ by $1-\alpha t$, so long as t is not high.

Relations (9) and (10), giving the specific gravities, are then expressed as follows:

$$a = a_0\frac{P}{p_0}(1-\alpha t) \tag{9'}$$

$$a' = a\frac{P'}{p}[1-\alpha(t'-t)]. \tag{10'}$$

Setting aside variations of pressure, it is seen that a rise of temperature of 1° diminishes the specific gravity by 1/273 of its value. In (9′) it is by 1/273 of its value at 0°, and in (10′) by 1/273 of its value at t'. The difference which has been noted, with regard to volumes, no longer exists. By applying the method of calculation to the volumes, moreover, the preceding difference is likewise done away with. We have thus a more exact idea of the degree of approximation realized by utilizing the method of calculation.

THE MIXTURE OF GASES.

Let us take the case of several masses of gas at the same temperature and occupying, respectively, volume v_1 under pressure p_1 volume v_2 under pressure p_2 for the second, and volume v_3 under pressure p_3 for the third, etc.

Let them be mixed in a capacity of volume V. The gases diffuse more or less rapidly, but in such a manner that the mixture finally becomes homogeneous. The temperature will not have changed if the gases have no chemical action on one another. The law of Dalton tells us that: The pressure of the mixture is equal to the sum of the pressures which the different masses of gas, made to occupy the volume of the mixture, would have.

The first gas would be subjected to the pressure $\frac{p_1v_1}{V}$, the second to a pressure of $\frac{p_2v_2}{V}$, etc.

If P be the pressure of the mixture, the law is expressed thus:

$$P = p_1\frac{v_1}{V} + p_2\frac{v_2}{V} + p_3\frac{v_3}{V}, \text{ etc.},$$

whence we deduce:

$$PV = p_1v_1 + p_2v_2 + p_3v_3, \text{ etc.}$$

If the two parts of the latter equation be divided by P, we shall obtain another enunciation of the law:

$$V = v_1\frac{p_1}{p} + v_2\frac{p_2}{p} + v_3\frac{p_3}{p}, \text{ etc.},$$

or: The volume of a mixture of several gases is equal to the sum of the volumes which the different masses of gas would have if the pressure of the mixture were brought to bear on them.

FIRST APPLICATION—SPECIFIC GRAVITY OF A MIXTURE OF TWO GASES.

This determination naturally supposes the specific gravities of the mixture and of the components to be considered under the same pressure, as, for instance, under pressure P of the mixture. Let V be the volume of the mixture, a its specific gravity: V_1, V_2, a_1, a_2 the volumes and the specific gravities of each of the components.

According to the second enunciation of the law, $V = V_1 V_2$.

The total gravity may be expressed in two ways. By equalizing the two expressions

$$Va = V_1a_1 \qquad V_2a_2;$$

whence

$$a = \frac{V_1}{V}a_1 \qquad \frac{V_2}{V}a_2.$$

V_1 and V_2 characterize the proportion in volume of each of the gases in the mixture. If we call the percentage of the first gas x, $\left(\frac{V_1}{V}\right)$, the percentage of the second will be

$$1 - x = \left(\frac{V_2}{V_1}\right).$$

The relation is expressed thus:

$$a = a_1x + a_2(1 - x).$$

We shall make use of this relation in determining the respective proportions of air and of hydrogen in an inflating gas of given specific gravity.

SECOND APPLICATION—GRAVITY OF A VOLUME OF MOIST AIR.

The quantity of water vapor contained in the atmosphere is expressed by the pressure which such water vapor would have, contained in a determined volume and supposed to occupy all that

volume alone. Such pressure is called tension of water vapor in air. It is generally denoted by the letter f. According to the law, the mass of dry air contained in that volume and made to occupy alone the whole volume would thus be $H-f$ at H pressure, H being the atmospheric pressure; that is, the pressure to which the mixture is subjected.

If the temperature of the moment be denoted by t; the weight of dry air contained in the volume is therefore equal to:

$$m = V \times 1.293 \times \frac{H-f}{760} \times \frac{1}{1+\alpha t}. \tag{11}$$

The specific gravity of water vapor contained in the same volume is equal to

$$m' = V \times 1.293 \times \tfrac{5}{8} \times \frac{f}{760} \times \frac{1}{1+t}. \tag{12}$$

$\frac{5}{8}$ being the density of water vapor.

The gravity of the volume V of moist air is thus equal to the sum

$$M = m + m'.$$

We may express it by taking as a factor the expression

$$V \times 1.293 \times \frac{1}{760} \times \frac{1}{1+t},$$

which we shall represent by X.

In consequence

$$M = X \times (H - f + \tfrac{5}{8} f) = X \times (H - \tfrac{3}{8} f).$$

Finally

$$M = V \times 1.923 \times \frac{H - \frac{3}{8} f}{760} \frac{1}{1+t} \tag{13}$$

this gives the gravity of a volume V of atmospherically moist air, H mm. of mercury f mm. of mercury and t degrees being the actual conditions of atmospheric pressure, of moisture, and of temperature.

It remains to specify how f is to be determined. It is the function of devices called "hygrometers." There is one of which the principle can not be contested, namely, that consisting of determining, by weighing, the gravity of water vapor contained in a volume of moist air after such water vapor has been absorbed by absorption tubes carefully calibrated. This amounts to direct measurement of the preceding gravity m'. In return, relation (12) gives f.

It is thus found that f, at a given temperature, is at most, equal to a certain value F, which is called maximum tension of the water vapor at that temperature. When such value has been attained, the water vapor commences to condense, to form small drops. Inversely,

if the tension of water vapor in a gaseous mass is lower than the saturation temperature, the water, brought into contact with the gas, vaporizes. At the end of a sufficient time, a gaseous mass in contact with water will thus be always saturated.

The maximum tension of water vapor continues to increase steadily with the temperature.

Herewith are stated some of the values:

t	10°	0°	10°	20°	25°	30°	40°	100°
F.....	*Mm.* 2.1	*Mm.* 4.6	*Mm.* 9.1	*Mm.* 17.4	*Mm.* 23.5	*Mm.* 31.5	*Mm.* 54.9	*Mm.* 760

The application of formula (12) gives us the gravities of water vapor contained in a cubic meter of gas at saturation tension.

It is found, at the same temperatures as those preceding, to be—

Gr. 2.3	*Gr.* 4.9	*Gr.* 9.3	*Gr.* 17.1	*Gr.* 23	*Gr.* 30	*Gr.* 50.6	*Gr.* 590

We shall find out precisely, by a numerical calculation, the respective value of the effects of temperature, of pressure, and of hygrometric conditions on the specific gravity of atmospheric air, expressed in the most general manner by the following:

$$\acute{a} = 1.293 \frac{H - \frac{3}{8} f}{760} \left(\frac{1}{1 + \alpha t} \right)$$

$$a = 1.293 \frac{H - \frac{3}{8} f}{760} (1 - t/273).$$

1. Let us first take the case of a variation of temperature alone, the other factors remaining the same. We shall notice that the fraction—

$$\frac{H - \frac{3}{8} f}{760}$$

when H is the atmospheric pressure, is very nearly 1. If we consider it as 1, it will be seen that the temperature being raised by 1°, the specific gravity diminishes by—

$$\frac{1.293 \text{ k.}}{273} \text{ or } 4.7 \text{ gr.}$$

If we take into consideration the limits within which the temperature varies as, for instance, by referring to the measurements given by an observatory (Montsouris, from 1871 to 1883), we find a maximum of 37° and a minimum of −23°; that is, a difference of 60°. The variations in specific gravity arising from the temperature alone thus amount to about 4.7 gr. × 60 = 280 gr.

2. Let us take the case in which the pressure alone varies, supposing the constant value of the other factors to have no practical effect on the value 1.293, an increase of pressure of 1 mm. of mercury increases the specific gravity by

$$\frac{1293}{760} = 1.7 \text{ gr.}$$

A variation of pressure of about 3 mm. of mercury has, therefore, the same influence as a variation of 1° in temperature.

The limits within which the pressure varies, according to quantities given by the Observatory of Montsouris from 1873 to 1884, are from 731 mm. to 787 mm. of mercury; that is, a difference of 56 mm., corresponding to a total variation of about 95 grammes.

3. In order to know the importance of the hygrometric conditions, let f be allowed to increase by 1 mm., when the specific gravity will diminish by about

$$\frac{1293}{760} \times \frac{3}{8}, \text{ or } 1.7 \text{ gr.} \times 3/8; \text{ that is, } .7 \text{ gr.}$$

There is, however, slight variation in the hygrometric condition. In the course of a day, even in summer, f rarely varies by more than 1 mm. The maximum scale of the variations does not attain more than 20 mm., so that the resulting maximum variation for specific gravity is 10 grammes.

The direction of the variation f is, moreover, the same on an average as that of the variations of t; so the lowering of the specific gravity of the atmospheric air, with the temperature, is slightly greater than that shown by the preceding quantities. This may be taken into account in the formulas by replacing the coefficient α by a rather higher figure. 1/250 or .004 are generally taken.

If we likewise take into consideration, in the gravity at 0° and 760, the slight quantity of water contained on an average in atmospheric air at that temperature, we shall be induced to adopt 1.292 as normal gravity. The gravity of a cubic meter of air would thus be given in terms of 2 variables only, by the formula—

$$a = 1.292 \frac{H}{760} (1 - .004\, t).$$

With an increase of 1° in temperature, the specific gravity would undergo a diminution of about $1.292 \times .004 = 5$ grammes.

The specific gravity of the air, for instance, at a temperature of 15° and with a pressure of 750 mm., would be equal to

$$1.292 - 5 \times 15 - 17 = 1.200 \text{ grammes.}$$

THE LIFTING POWER OF A GAS.

The idea of the lifting power of a gas, which is fundamental in ballooning, is the result of the application of the principle of Archimedes to the unit of volume; that is, to the cubic meter of light gas immersed in air, by abstracting the envelope which contains that gas. The two forces acting upon it are, therefore, its weight b downwards, and the thrust of Archimedes a upwards. a and b are nothing more than the specific gravities of the air and of the gas. The resultant of these forces, directed upwards, is equivalent to $a-b$.

It is this difference which is called the lifting power of gas.

The lifting force of a gas, however light and rarefied, is thus, at most, equal to a.

Its value varies, moreover, with the conditions of pressure and temperature, like the two specific gravities which enter into its expression.

With a view to simplicity, variations in atmospheric moisture will not be taken into account. The specific gravity a is thus expressed:

$$a = a_0 \frac{P}{760} \frac{1}{1+\alpha t}$$

There is no hypothesis to be made for the moment with regard to the water vapor contained in gas. It should be considered as taking a part similar to that of any ordinary impurity contained in the gas, such as air, for instance, which only intervenes by increasing the average specific gravity. Let b_0 be the specific gravity of the gas at 0° and at 760. If the gas be of the same pressure p and the same temperature t as the exterior air, we get:

$$b = b_0 \frac{P}{760} \frac{1}{1+\alpha t}$$

Therefore:

$$a - b = (a_0 - b_0) \frac{P}{760} \frac{1}{1+\alpha t}$$

Lifting power is said to follow the laws of Mariotte and of Gay-Lussac.

It must be remarked that the gas contained in a balloon is generally under conditions of temperature and pressure different to those of the outer air.

In a dirigible, for instance, the gas is subjected to an interior excessive pressure, the average value of which may be taken as equal to 40 mm. of water or 3 mm. of mercury. The specific gravity of the gas then increases by 3/760 of its value, which is about $\frac{1}{2}$ gramme for an average inflation gas. The lifting power thus diminishes

correspondingly. For a balloon of 6000 cubic meters, like the *Fleurus*, such excess of pressure is equivalent to an overweight of

$$6000 \times .5 \text{ gr.} = 3 \text{ kgs.}$$

which is very low.

The temperature is, on the other hand, generally higher in climbing than the outer temperature:

$$t' > t.$$

Supposing that

$$t' = t + \theta.$$

θ is what is called the heating.

The specific gravity b becomes:

$$b_0 \frac{P}{760} \frac{1}{1+\alpha(t+\theta)}$$

or

$$b_0 \frac{P}{760} \left[1 - \alpha(t+\theta) \right]$$

instead of:

$$b_0 \frac{P}{760} \ (1 - \alpha t)$$

It has thus diminished by:

$$b_0 \frac{P}{760} \alpha\theta$$

The lifting power has therefore increased to the same extent. Let $P/760$ be considered as 1. The lifting power of the gas thus increases, per degree of heating, by:

$$\frac{b_0}{273}$$

that is, about ½ gramme for an average gas.

When a dirigible is moving, the heating does not generally attain high value, on account of the continual cooling of the streamline by the wind of its own speed.

In a free balloon, on the contrary, it frequently happens that the temperature of the interior of the balloon is 20° or 30° higher than that of the outer temperature. In the case of a balloon with a capacity of 750 cubic meters, the lightening thus produced would be about .5 gr. $\times 750 \times 25$; that is, nearly 10 kilos, which is very remarkable in relation to the quantity of maneuvering ballast carried by such a balloon. Proportionate lightenings may even be produced in a dirigible when the wind of its own speed is diminished or stopped by slowing down or stopping the propellers, as, for instance, when landing.

The purity of a gas.—In order to know the aerostatic value of a gas of which the lifting power is known, it is essential that the conditions of temperature and of pressure, corresponding to the quantity given, are precisely known. As the nominal lifting power of a gas, the quantity is always taken which supposes this gas to be brought under normal conditions (0° – 760 mm.), and it is specified as being in ratio to normal dry air.

The number thus given is characteristic of the purity of the gas.

Take, for instance, the quantity of 1.155 grammes found on the *Fleurus*, March 16, 1917.

We get:

$$a_0 - b_0 = 1293 - b_0 = 1155.$$

Therefore

$$b_0 = 1293 - 1155 = 138.$$

Suppose, for the sake of simplicity, that the only impurity contained is air.

We shall apply the formula of the mixture:

$$a = a_1 x + a_2 \ (1 - x)$$

where we ought to make $a_1 = 89$, specific gravity of pure hydrogen and

$$a_2 = 1293, \text{ specific weight of air.}$$

We get

$$138 = 89x1293(1 - x),$$

$$(1293 - 89)x = 1293 - 138 = 1155.$$

Therefore

$$x = \frac{1155}{1204} = 95 \text{ per cent.}$$

1204 is, however, the lifting power of pure hydrogen. It is practically the maximum value of the nominal lifting power.

The percentage of pure hydrogen of a gas is thus equal to the ratio of the lifting power of the gas with the lifting power of pure *H*.

In the case of fresh gas, collected at the time of inflation, at the delivery pipe of the factory, the lifting power attains and exceeds 1185. The proportion of pure *H* contained is thus about:

$$\frac{1185}{1204} = 98 \text{ per cent.}$$

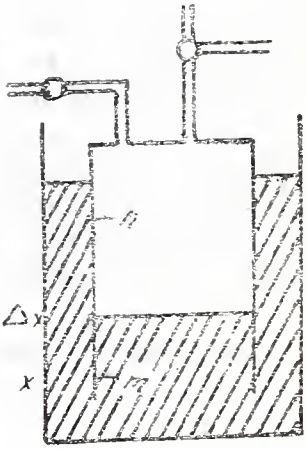
NOTE.—It must not be forgotten that the lifting power of a gas, as we have just defined it and examined it, relates to a determined volume of gas (the cubic meter), but to a variable mass. Moreover, the total lifting power of a determined mass of gas is evidently constant.

Measurement of lifting power.—It is based on the application of the following law of Bernouilli:

Two equal volumes of two gases, under the same pressure, take time proportional to the square roots of their densities, in flowing through a capillary opening.

The measurement is realized by the Schilling device, which is composed of a test bar, turned upside down, and a tub containing oil or water.

Two indicating marks, m and n, are made on the test bar; they limit the gaseous volume which is made to flow through the capillary hole. The pressure which determines the flow of the gas is the height of the column of oil separating the two levels in the interior and on the exterior of the test bar. The pressure is thus variable during the operation, and the wisdom of the application of the law may be doubted. The following argument cancels the doubt:



Let us consider, for hydrogen and for air, the passage of the oil column of a height x at the height $x+\triangle x$, $\triangle x$ being sufficiently small to enable us to consider the flow of gas taking place in that interval to be under constant average pressure. If $\triangle t$ and $\triangle T$ represent the time taken by hydrogen and by air, d and D being their respective densities, we get, according to the law:

$$\triangle t=\sqrt{\frac{d}{D}}\times\triangle T.$$

Thus, in making up the sum of these elementary quantities of time, t and T remain connected by the same law of proportionality.

$$t=\sqrt{\frac{d}{D}}\,T.$$

From which we shall deduce d density of the hydrogen.

As a matter of fact, D, the density of the air, is equal to 1. If it is squared, the result will be:

$$d=\frac{t^2}{T^2}$$

The specific gravity of the gas at 0° and at 760 is therefore expressed as follows:

$$b_0=1.293\times\frac{t^2}{T^2}$$

and the lifting power in ratio to the dry air, as previously specified, is equal to:

$$1.293 - b_0$$

It is thus stated

$$\varphi_0 = 1.293 \left(1 - \frac{t^2}{T^2}\right)$$

Such is the formula for the use of the Schilling device. In order to simplify calculations, it is convenient to have a table in which the lifting power is given in terms of $\frac{t}{T}$. The operations are thus reduced to 1 division. By means of a table of double entry, or more simply by an abacus, any operation is done away with; by simply reading it, the lifting power may be found in terms of t and T.

NOTE.—D has been taken to equal 1. But the air which passes through the apparatus, and which is atmospheric air, always contains a small quantity of steam. Such as it is, it is lighter than dry air under the same conditions of temperature and pressure. D is thus a little smaller than 1.

Thus $d = D \frac{t^2}{T^2}$ is smaller, in reality, than the figure adopted $\frac{t^2}{T^2}$.

Thus

$$b_0 < 1.293 \frac{t^2}{T^2}$$

The lifting power is therefore greater than the given quantity. Let us examine the extent of the error committed. We have previously seen that 10 grammes is the maximum difference of specific gravity resulting from water vapor in the air. The smallest value or D is therefore

$$\frac{1.283}{1.293} = 1 - \frac{10}{1.293}$$

The difference which results between the true density and the quantity adopted is

$$\frac{10}{1.293} \frac{t^2}{T^2}.$$

For specific gravity or for lifting power, this maximum difference is therefore

$$1.293 \times \frac{10t^2}{1.293\,T^2}$$

or

$$10 \frac{t^2}{T^2},$$

that is, 1 gramme, since $\frac{t^2}{T^2}$ is very near 1/10.

The maximum systematic error realized by taking the density of atmospheric air equal to 1 is to the extent of 1 gramme. Under average conditions, it varies from 3 to 6 decigrammes. As the measure does not include that precise amount there is no need of correction for that reason. As for the water vapor contained in the gas, it must be recollected that it is an impurity like all others.

Precision of the measurement.—The result is expressed within 1 gramme. It would be an illusion to give a result with a precision which does not exist in the measurement. The operation must therefore be conducted with such precautions that the error in the lifting power is not more than 1 gramme. We have seen that this error is complete in the term $b_0 = 1.293 \times \frac{t^2}{T^2}$.

The extent of $\frac{t^2}{T^2}$ being 1/10, the extent of b_0 is 130 grammes. An error of 1 gramme would constitute a relative error of about 1/100. The relative error in $\frac{t^2}{T^2}$ being equal to twice the relative error in t and more than twice the relative error in T, it is thus necessary that:

$$z\frac{\triangle t}{t} + z\frac{\triangle T}{T} \leqslant \frac{1}{100}$$

or

$$\frac{\triangle t}{t} + \frac{\triangle T}{T} < \frac{1}{200}$$

In the device used in the *Fleurus*, the duration of the flow of hydrogen is 50 seconds. It is measured by means of a second meter which gives 1/5 of seconds. If it be granted that a mistake may be made in less than 1/5 of a second, the relative error in t is therefore $<$ 1/250.

T is the amount of 150 seconds. An absolute error of 1/5 of a second corresponds to a relative error of 1/750.

But $1/250 + 1/750 = 4/750$, which is a little larger than 1/200. The exactitude thus obtained is therefore scarcely sufficient.

It is, consequently, absolutely necessary that an error greater than 1/5 of a second should not be made in the measurements of time. The operation should thus be carried out with care. It is essential that the following precautions be taken into account: For each time, several readings should be made, and they should coincide as far as possible. If they do not coincide, the average should be taken, and it should not deviate by more than 1/5 of a second from the highest numbers stated. Ebullitions, which might leave gaseous bubbles adhering to the walls of the test-tube, should be avoided. The gauge should be cleansed many times when it is filled

with hydrogen after having contained air, or the reverse. Ten cleanings are only just sufficient in the former case. To avoid, on the other hand, air infiltration during the intake of hydrogen, by the joints, couplings, manometric pipes, etc., lifting power should only be taken if the excess of pressure in the interior of the dirigible (or of the auxiliary tank) be at least equal to 15 millimeters of water. Such excess of pressure has no effect on the measurement, since the pressure of the flow of gas through the capillary opening depends only on the distance between the two marks.

Finally, for a series of measurements, such as those of which the result is daily inscribed on the upkeep records, they must be taken by the same operator, each one having his own manner of making the release or the stoppage of the chronometer tally with the passage of the liquid meniscus before the two marks on the gauge.

Results.—A quantity is thus found which diminishes in proportion to the duration of the inflation of a balloon; it decreases so much more slowly when the envelope of the balloon is more impermeable and when the upkeep of the balloon is carried out under the best conditions. This variation, it may well be understood, has no relation whatever to the variation caused by changes of temperature or pressure.

Example.—The lifting power of the gas of *La Flandre,* at the time of inflation, was as 1.169; after 2 months, 1.149; after 4 months, 1.137.

In consequence, the replacement of a considerable part of the balloon gas by fresh gas produced the result of raising the lifting power. *La Champagne,* for instance, having made a war ascent February 23, 1917, in the course of which 6,000 cubic meters of gas had to be exhausted in order to climb about 4,200 meters, the reinflation of the balloon on the following day raised its lifting power from 1.166 to 1.174. A balloon like the *Fleurus,* which often goes up, and the envelope of which is of good quality, does not lower its lifting power. In September, 1916, it was 1.153; in March, 1917, it was 1.155.

The lifting power is generally found to be lower in summer. Among the causes of this lowering, there is the increase in the quantity of water vapor contained in the inflating gas. We shall see the quantitative effect of the moisture contained in the gas with a tension of f steam.

The specific gravity of the inflating gas is denoted by b.

Let the specific gravity of the same gas, supposed to be dry, be denoted by b (right), and brought under the same conditions of temperature and pressure.

We shall apply the law of the mixture of gases as for a weight of moist air.

= the sum of the gravities of dry gas and moisture contained in a cubic meter:

$$= b_0 \frac{p-f}{760} \times \frac{1}{1+\alpha t} + 1.293 \times 5/8 \times f/760 \times \frac{1}{1+\alpha t} =$$

$$b_0 \underbrace{\frac{p}{760} \frac{1}{1+\alpha t}}_{b} + \frac{f}{760} \frac{1}{1+\alpha t} (800 - b_0)$$

The first term may be supposed equal to b (right). The second term shows the increase of specific gravity due to the presence of moisture. In order to estimate its average numerical importance, we shall take b_0 to equal about 125, taking no account of the difference between the factor $\frac{1}{1+\alpha t}$. The corrective term is thus equal to:

$$f \frac{675}{760} = f \times .8$$

The specific gravity of the inflating gas is thus increased by about .8 gr. per millimeter of tension of the water vapor contained. But the balloon in the hangar, connected with the auxiliary tank and with the pipes leading from the factory, which contain water, forms a gaseous receiver in the presence of liquid water; its gas is therefore saturated with moisture. The tension of saturation realized in the receiver is moreover defined by the lowest temperature of the receiver (principle of the cold wall). If, in consequence, a considerable length of the connecting pipe is underground, its low temperature in summer will cause a lower tension of water vapor in the balloon than the saturation tension corresponding to the temperature of the atmosphere or of the hangar. If, on the contrary, the connecting pipe issuing from the factory is short, if the gas is injected direct from the works into the balloon, and the water used to cool it was not of sufficiently low temperature, there will be a considerable quantity of water vapor contained in it. Under these conditions, the lifting power is lowered by 20 grammes or thereabouts.

Lifting power of gases other than hydrogen.—We shall now make a note of the average values of the lifting forces of the two other gases which have been used in ballooning.

Illuminating gas is still sometimes used in inflating balloons. We have seen that its specific gravity is about 520 grammes per cubic meter. Its lifting power would thus be about 770 grammes. Taking into account the air mixed with the gas at the time of inflation, a lifting power far beyond 700 grammes should never be reckoned upon. It is only under special circumstances, as, for example, when a company could be induced to drive the gas produced in the

last stage of distillation of the coal directly into the balloon; such gas being appreciably richer in H, a higher lifting power might be advantageously obtained, which might even exceed 900 grammes.

There is no longer any question of inflating balloons with hot air. It might, however, be of interest to know the value of that inflating gas, if only for the purpose of becoming better acquainted with the means by which the first balloonists accomplished their renowned exploits and opened up a perspective of further realizations. In reality, the hot air with which a Montgolfier is filled is not air alone; it contains a large proportion of the products of the combustion of straw or other substance which was burned in its appendix. Among these products, carbonic gas and steam head the list. The former being heavier than air, the latter lighter, and consequently the more advantageous. Its production may be increased, for instance, by burning damp straw, the water of which vaporizes and is added to the combustion gases and to the column of hot air engendered by them. Let it be granted that the inflation gas thus obtained has an average density nearly approaching that of the air. Its weight per cubic meter will therefore be about $1.290 \times 1/1 + \alpha t$; t being the temperature in the interior of the balloon. t never exceeds 100 degrees, and a higher temperature is moreover highly detrimental to the impermeability and preservation of the fabrics. We shall therefore adopt that figure.

On the other hand, let 15° be an average quantity for the atmospheric temperature. The lifting power of the cubic meter of gas is thus, no account being taken of the difference between the normal pressure and the pressure at the time:

$$\frac{1.290 \times 1}{1 + \frac{15}{273}} = \frac{1.290 \times 1}{1 + \frac{100}{273}}$$

When all these operations have been made, about 260 grammes are obtained.

In practice, only 200 grammes per cubic meter of gas can be calculated upon. The dimensions of the first Montgolfiers were therefore considerable in comparison to the free balloons of the present day. The first balloon to rise in a free climb, November 21, 1783, mounted by the Marquis d'Arlandes and Pilatre de Rozier, measured 20 meters in diameter and 16 meters in height. The volume should, therefore, have been about 8,000 cubic meters. Two months after, the fourth free climb took place at Lyons; the balloon *Le Flesselles*, which made the climb and had Montgolfier himself on board with six other passengers, measured 42 meters high and 35 meters in diameter. Its volume exceeded 50,000 cubic meters.

THE WEIGHING OF A BALLOON.

A knowledge of the lifting power of the gas enables us to determine the utilizable value of the balloon or the load that it might raise. It suffices to write the equation of equilibrium of the balloon.

We shall define the forces acting upon it: The weight from top to bottom, the gaseous mass and the solid parts. If V be the volume of the gaseous mass, its weight is expressed as Vb. Let P represent the total weight of the solid parts, in the sense generally given to the weight of a solid body, that is, irrespective of its weight in the air.

The thrust of Archimedes on the volume of the gaseous mass is therefore Va; the thrust of Archimedes on the solid parts are subtracted in P.

The equation of equilibrium is written thus:

$$Va = Vb + P$$

or

$$V(a-b) = P. \tag{1}$$

We have just stated that P may be calculated when the lifting power is known.

The operation of weighing consists, on the other hand, in determining experimentally the load raised. The result obtained should therefore agree with the number obtained in consequence of the determination of the lifting power. The two operations may be utilized in controlling one another.

It can be proved by the first part of equation (1) that the lifting power varies according to the law of Mariotte-Gay-Lussac, and in the second part that the total solid weight P consists of an invariable part π which includes the weight of the envelope, of the suspension, the basket, the instruments and tackle, tools and spares, engines in working order and of a remainder U which includes the passengers, the offensive and defensive loading, the sand or water ballast, reserve water for the radiators, fuel and lubricating substances.

U is called the useful load or weight.

The equation becomes:

$$V\varphi_0 \frac{P}{760} \frac{1}{1+\alpha t} = \pi + U. \tag{2}$$

The relative importance of the three terms, for the *Fleurus*, for instance, by adopting $V = 6.500$ and $\varphi_0 = 1.150$, is shown by the following quantities: The first part is 7.500 kilos; the solid invariable weight is about 5,000 kilos; the useful load is therefore 2.500 kilos.

It is thus seen how much the variations of temperature and pressure affect this useful load, since it alone is actually subjected to the variations caused in the first part by variations of t and p. These

variations are here about three times as great as the proportion of Mariotte-Gay-Lussac, applied directly to the number U, would appear to indicate.

It is consequently important, as for lifting power and even more so, to reduce the quantity obtained for the useful load to determined conditions of temperature and pressure. We might adopt, for instance, 15.760.

In making the approximations taken up to the present to show up in succession the part which amounts, in the variations of U, to each of the factors p and t, it is seen that an increase of 1° of temperature produces a diminution of $\frac{V\varphi_0}{273}$ in U; in the same way, an increase of 1 millimeter of mercury produces an increase of U equal to $\frac{V\varphi_0}{760}$.

In the case of the *Fleurus*, the quantities utilized in making corrections are 28 kilograms per degree of temperature and 10 kilograms per millimeter of pressure, the direction of these corrections resulting from what precedes.

NOTE.—The interior pressure likewise enters into consideration in modifying the weight. In fact, the volume of a flexible balloon is not strictly constant. The effect of the increase of interior pressure is to diminish the ovalization of its section. The latter, in consequence, increases its surface as it becomes circular in shape. The total volume of the balloon thus increases. By way of example, the variations of volume experienced by the *Fleurus* for successive increased pressures of 1 millimeter of water are herewith shown. q denotes the interior increasing excessive pressure read by the manometer of the basket; ΔV are the variations of corresponding volume. These variations of volume are sufficiently important to be taken into account. In fact, the increase of useful load due to excessive interior pressure, which varies from 10 to 20 millimeters in the case of the *Fleurus*, exceeds 30 kilos.

Finally, the nominal weight is reduced to 760 millimeters and 15 millimeters of excessive interior pressure. It is deduced from weight determined under conditions t, p, and q by the following statement:

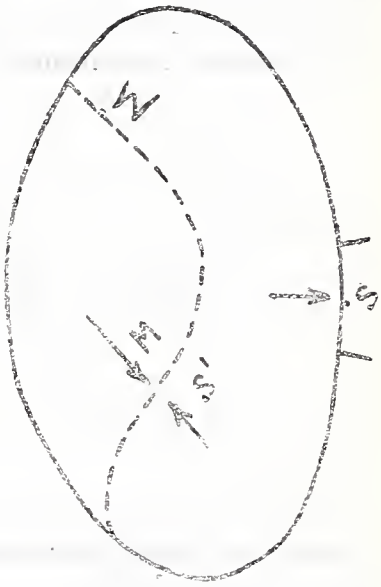
$$U(.760,\ 15mm) = U(t,\ p,\ q) + \varphi_0\left[\frac{V}{273}t + \frac{V}{760}(760-p) + \Delta V\right]$$

V being the volume of the carene under the pressure of 15 millimeters and V being the increase of volume when the excessive interior pressure passes from q millimeters to 15 millimeters.

q	ΔV	q	ΔV
Millimeters.	*Cubic meters.*	*Millimeters.*	*Cubic meters.*
1	12	13	.....
2	10	14	3
3	8	15	3
4	7	16	3
5	6	17	3
6	4	18	2
7	4	19	2
8	4	20	2
9	4	21	2
10	4	22	2
11	4	23	2
12	3	24	1
13	.....	25	1

PASCAL'S PRINCIPLE—THE INTERIOR PRESSURE OF BALLOONS.

Pressure at one point of a fluid.—Let us take the case of a heavy fluid in a state of equilibrium, contained in a receiver, the interior wall of which is Σ; suppose s to be an element of the wall small enough to be considered as an element of the plane. The element s is subjected, on the part of the fluid, to a normal pressure directed from the interior to the exterior of the fluid. This pressure may be made evident and may even be measured if the element of the wall s is movable like the piston of a cylinder. It is found to be independent of the orientation given to s round the same average point.



Take the case of an element of surface s' in the interior of the fluid. It is subjected to normal pressure on its two surfaces; as a matter of fact, s' may be supposed to form part of a surface Σ, the existence of which would bring about no change in the equilibrium of the gaseous mass, and which would limit on either side two masses of fluid; each of these would exert normal pressure on s' from the interior toward the exterior.

Such pressures, exerted on each surface of s', may be expected to be equal since the orientation of the element of the wall does not reduce the force; this is shown by experiment, the element s' actually remaining in equilibrium. The value of the force brought to bear on the unit of surface, independent of the orientation of the element round an average point M is called the "pressure of the fluid at that point." It is therefore the quotient of the force F and the element of surface on which it is exerted $P = \frac{F}{s}$.

Laws of the statics of heavy fluids.—First law: The pressure is the same at all the points of a horizontal plane. Second law: The difference between the pressures at two points A and B of a certain fluid mass is equal to the weight of a fluid column having the unit of surface as a base, and the vertical distance between the two horizontal planes passing through A and B as their height. If it be granted that the specific gravity of the fluid be considered as constant between A and B and equal to a, the result would be:

$$P_1 - P_2 = ha.$$

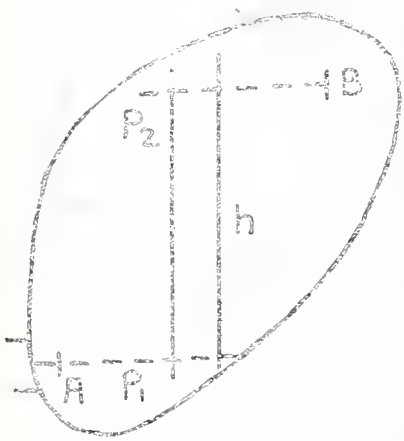
If this hypothesis can not be granted, the weight of the fluid column may still be put into the form $\pi = ha$. This would give:

$$P_1 - P_2 = \pi = ha$$

and a will be the average specific gravity of the fluid.

This fundamental theorem of the statics of fluids is called the principle of Pascal.

Sometimes, however, this name has been reserved for a corollary of the theorem, resultant of its application to the particular case of noncompressible fluids, as is practically the case for liquids; their specific gravity is therefore independent of variations of pressure. Suppose in such a case that the pressure is increased, say at level 1, by working at a piston placed on a wall; the difference between p_1 and p_2 maintains the same value, a not having varied. p_2 has thus undergone the same increase as p_1; and it is the same with the pressure at all points of the fluid. It is said that liquids transmit pressures (principle of Pascal, according to some authors).



This transmission of pressures is not the case with gases.

INTERIOR PRESSURE.

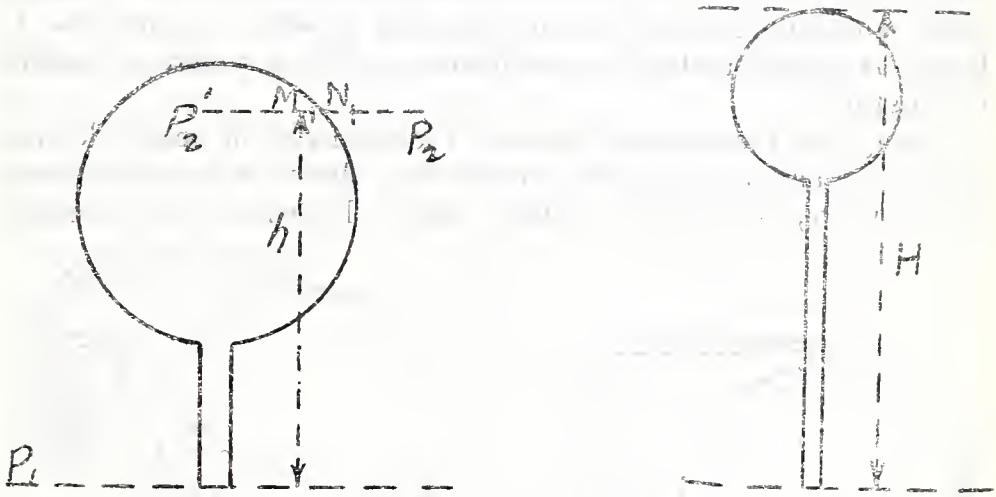
Application of the fundamental theorem.—Let us take the case of a spherical balloon equipped with an appendix sleeve open in its lower part and filled with light gas down to the bottom of the appendix sleeve.

The balloon is immersed in air in a state of equilibrium, and it remains in equilibrium so long as the temperature and exterior pressure do not vary. At the mouth of the appendix sleeve the two fluids are in contact and superposed in the order of their density; this section fulfills the same function as the free surface of a fluid

heavier than air, such as a liquid. Two points of the section, closely adjacent, one in the air, the other in the inflating gas, are at the same pressure; this section is said to constitute a plane of equal pressure for the two fluids.

Suppose p_1 to be this common pressure.

In a plane situated at a height h above the former, we shall consider a point situated in the interior of the balloon and another on its exterior, such as two points on either side of the envelope, M and



N. Let p_2 be the pressure on the exterior at N, and p'_2 the pressure in the interior, at M.

We shall apply the principle of Pascal twice, by rising with each gas.

In the air:

$$p_1 - p_2 = h.a \text{ (a being the average specific gravity of the air).}$$

In the case of the inflating gas:

$$p_1 - p_2 = h\,b \text{ (b being the average specific gravity of the gas.)}$$

By eliminating one factor in each equation it follows that—

$$\begin{aligned} p'_2 - p_2 &= h(a-b) \\ &= h\varphi \end{aligned}$$

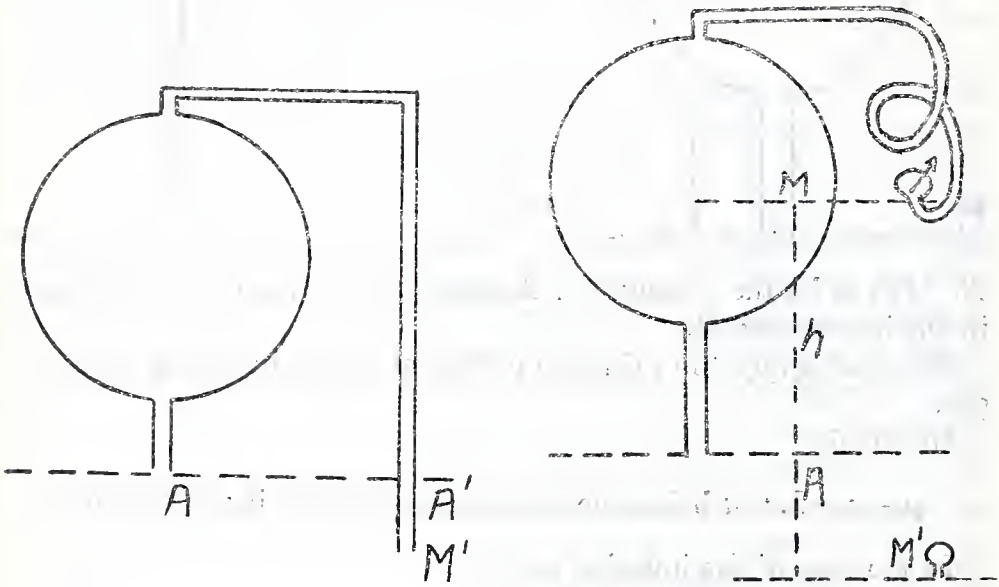
the lifting power of the gas being denoted, as before, by φ.

The pressure in the interior of the balloon is therefore greater than the pressure on its exterior. To such difference the name of "apparent pressure" or "abnormal pressure" is given; or sometimes it is called interior pressure.

It is always estimated in kilograms per square meter or—which comes to the same thing—in millimeters of water.

The interior pressure is thus so much greater when the point taken is raised so much higher than the plane of equal pressure, and in proportion to the lightness of the balloon gas. In the case of a given gas, the mass of which is continuous, the vertical distance alone, above the plane of equal pressure, is taken into account and not the form of the receiver containing the gas. A balloon with a straight and very long sleeve would admit of an experiment similar to that of Pascal's tub, with respect to liquids; if a balloon of that kind were filled to the bottom of the sleeve, which will be filled with a very slight quantity of gas, a high abnormal pressure proportional to height H will be realized in the balloon, which it would be capable of bursting.

Example of the normal balloon.—The diameter of the balloon is 10 meters and its appendix sleeve (free climb) is 2 meters long. At the valve, located at the upper pole of the balloon, the excessive



pressure is thus equal to $12\times$, that is, about 13 millimeters of water for a balloon inflated with hydrogen; this would be shown by a manometer placed at that point.

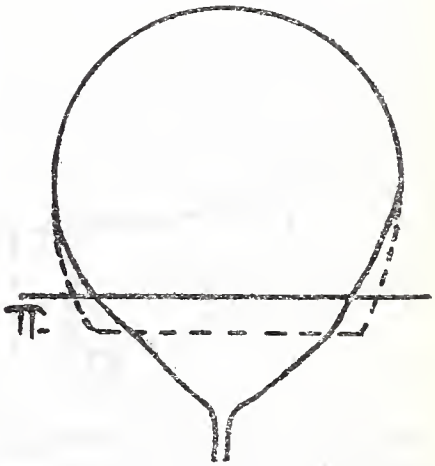
If the manometer is coupled at that point, however, or if it is placed on a lower level, it must be noticed that the tubular connected with the manometer forms part of the receiver of the gas and that the shape of the receiver is of no importance. The manometer will indicate the excessive pressure existing in the plane where it is situated itself and which corresponds to the height $A\ M$ above the plane of equal pressure.

If the length of the pipe is sufficient and the manometer is lowered to M' below the sleeve of the appendix without disconnecting it, that is, by maintaining the continuity of the gaseous mass in the interior, the application of Pascal's principle shows at the level M'

a pressure in the gas lower than that of the air. In fact, the manometer indicates a depression. If the manometer be disconnected, the level of gas in the pipe takes up the same plane in the sleeve of the appendix as in the communicating vases. The end A' M' of the pipe is filled with air at the exterior pressure; the manometer, newly connected, shows no indication.

A liquid transmission might be imagined, on the contrary, by means of which the pressure existing at the point at which the piping reaches the balloon might be read in the nacelle.

Slack balloon.—When the exterior pressure begins to increase, for instance, when a balloon descends to an altitude lower than that already attained or if the temperature has just fallen, the sleeve of the appendix is found to flatten and close, while the fabric makes folds and has a tendency to rise in the lower part, whereas it remains taut in the upper part of the balloon. From this it may be concluded that the pressure in the interior of the balloon remains greater in the upper part than the exterior pressure and that it becomes less in the lower part. There is thus in some part a plane π of equal pressure.



At each point the apparent pressure is equal to $h\varphi$, h being the vertical distance from the point of plane π, counted with its sign positively toward the top, negatively toward the bottom.

If the fabric were not heavy or had no stiffness of its own, the balloon would have the shape of a truncated sphere, as shown by the dotted lines, instead of being of an undulating pear shape as fully outlined. The excessive exterior pressure would bring the envelope against the plane representing the free surface of the gas, which would thus be the plane of equal pressure. This plane would be below plane π, and the pressure on the upper part of the balloon would therefore be greater.

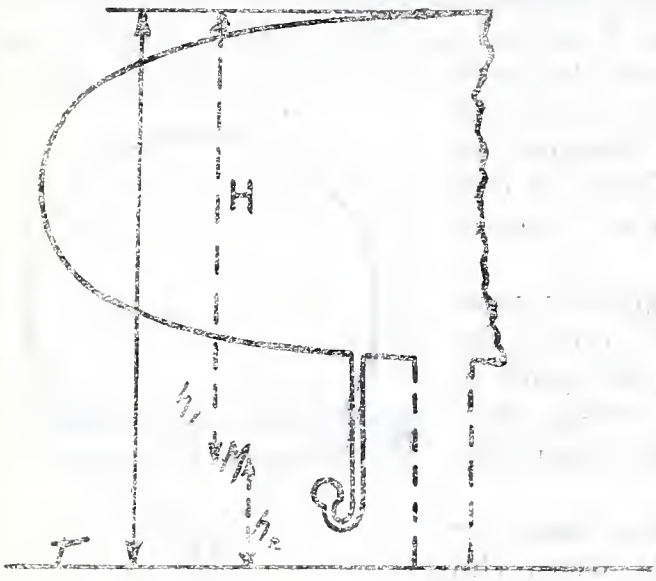
In keeping a spherical balloon in the hangar, it should be placed in the position shown in the preceding dotted outline by attaching the ballast bags which keep the balloon on the canvas of the hangar, at suitable points.

The slack part of the envelope, by means of which the excess of exterior pressure would create important air infiltration, is thus done away with; at the same time the pressure in the upper part of the balloon is increased. Such increase is, moreover, the more considerable in proportion to the weight of the ballast bags.

THE CASE OF DIRIGIBLES.

Interior pressure in dirigibles.—In a flexible dirigible, the interior abnormal pressure should be maintained in order to obtain that permanence of shape necessary in navigation at a higher value than the pressures attained in spherical balloons, captive or free. It must be remarked that in captive balloons a pressure greater than that of free balloons is engendered by the adoption of a longer appendix sleeve.

The maneuver and upkeep of the dirigible balloon would be inconvenienced by a sleeve open to the air; it would necessarily be stiff and would extend well below the nacelle. Its only advantage would be that of limiting automatically the excessive pressure at maximum value, which would depend on the length selected for the sleeve; its inconveniences are too glaring to need notification.



The balloon is closed; the limitation of the excessive pressure is brought about by means of the exhaustion of gas by controlled valves, which are also capable of working automatically above the corresponding pressure at a chosen calibration.

The plane of equal pressure pushed below the nacelle is thus fictitious. It is that in which the open sleeve ends. The manometer situated at M in the nacelle thus indicates the pressure $h_0\varphi$, h_0 being the height of the manometer above plane π.

In the upper part of the balloon the pressure is $h_1\varphi$.

If q is therefore the pressure read by the manometer of the nacelle the pressure in the upper part is equal to

$$q + (h_1 - h_0)\varphi$$

or

$$q + H\varphi$$

H being the height of the balloon.

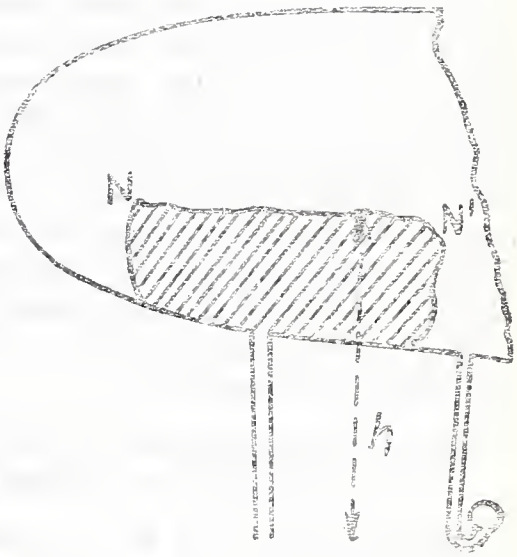
Let it be remarked that φ nearly 1.150 kilogrammes is not very different from 1. It may thus be said on the whole that the pressure at any point whatever of the balloon is equal to the pressure read in

the nacelle, increased by as many millimeters of water as the point measures in meters of height above the nacelle.

The pressure of navigation read on the manometer of the nacelle varies according to the types of French balloons from about 15 to 40; this therefore gives maximum pressure of 30 to 60 at the upper point of the balloon, and pressures of about 20 to 45 millimeters in the lower part of it.

Pressure in the ballonets.—Flexible dirigibles are obligatorily equipped with one or several ballonets, into which air may be driven to keep up the interior excess of pressure.

Let us take the case of a ballonet of this kind, inclosing a certain quantity of air. Let a' be the average specific gravity of that air, slightly compressed, a being the specific gravity of the exterior air $a' > a$.



When the mass of air in the ballonet is raised by 1 meter, the pressure diminishes by a'; when there is a rise of 1 meter in the outer air the pressure diminishes by a.

The pressure in the ballonet thus decreases rather more quickly; the overpressure in the ballonet consequently lowers from a' to a per meter cleared upwards.

If the overpressure is of the average value of 40 millimeters of water, for instance, the application of the law of Mariotte gives a low value of about 5 grammes for $a' - a$.

In order that the overpressure may be lowered only by one-half millimeter of water or 500 grammes per square meter, it would be necessary to clear an interval in height of 100 meters.

As the height of the ballonet and of its piping would only come to about one-tenth of that interval, it may be granted that the overpressure is the same in the whole mass of air in the ballonet. The overpressure read at the car, at the lower end of the piping, is thus the same as at any point of the ballonet itself.

In order to know what this overpressure is, let us consider what is the upper level N reached by the air, which being denser than hydrogen of inflation, first fills the bottom of the ballonet and then rises gradually. The upper level is a plane of equal pressure between the air of the ballonet and the inflation gas.

Theoretically, the upper wall NN' of the ballonet might be suppressed, and the two gases would remain in equilibrium and in contact on that plane, superposed in the order of their density. The interposition of the envelope of the ballonet only increases the pressure in the interior of the ballonet of the weight per square meter of that envelope; that is, by some 300 or 400 gr., that influence might therefore be neglected as unimportant.

The overpressure in the upper level of the mass of air in the ballonet, and consequently in all that mass of air, is thus equal to the overpressure in the mass of hydrogen at the height NN' of the plane; it is as follows:

$$q + \varphi h$$

if q be the pressure shown at the car by the manometer in communication with the hydrogen. As soon as the first quantities of air are driven into the ballonet, the pressure there immediately becomes higher than that indicated by the gas manometer. The pressure increases in proportion to the new quantities of air driven in; that is, the level rises in the ballonet. To a certain extent, the pressure read on the manometer may thus give some idea of the total quantity of air driven in, and an indication may be thus found for preferring to ventilate in the front or in the rear of the ballonet. Other rules for maneuvering, decided by other considerations, give closer indications on the subject.

SLIGHT DISPLACEMENT ON THE VERTICAL—CORRESPONDING VARIATION OF THE INTERIOR PRESSURE.

The dirigible balloon being closed, let us suppose for a moment that its volume is constant, that is, that the balloon may be nondeformable. The pressure of the gas (not its apparent pressure) is thus constant while the gaseous mass remains the same. Suppose the balloon to rise 10 meters, for instance; the exterior pressure is lowered by $10 \times a$, that is, 10×1.250 kilog. = about 12 kilogrammes. The interior pressure has not varied; therefore the apparent pressure or interior overpressure would vary in proportion to the exterior pressure by 12 millimeters of water.

If flexible balloons were constant in volume, the interior pressure would therefore vary rapidly in the case of a slight displacement on the vertical.

We have already seen that this is not the case, and we have given the table of variations of volume of the *Fleurus*. Knowing this table of variations, the variation of apparent pressure for an elevation of 10 meters, for instance, might be anticipated by approximate calculation.

The constant mass of gas in the interior of the balloon undergoes a diminution of pressure $\triangle p$, which is the diminution of exterior pressure. Its volume thus increases by $\triangle v$, so that

$$v \triangle p = p \triangle v$$

$$\triangle v = v \frac{\triangle p}{p}$$

We shall see that in the case of an elevation of 10 meters there is about

$$\frac{\triangle p}{p} = 1/800.$$

In the case of the *Fleurus*, therefore,

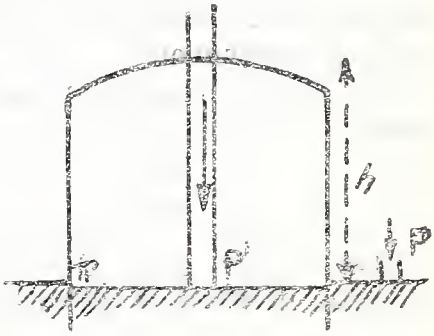
$$\triangle v = \frac{6500}{800} = 8 \text{ cubic meters (about).}$$

By referring to the table, we see that if the initial interior pressure were 20 millimeters, it would be 23 millimeters for an increase of volume of 8 cubic meters. This result agrees with that observed in climbing.

AUXILIARY SUPPLY BALLOONS.

PRESSURE IN THE INTERIOR OF AN AUXILIARY SUPPLY BALLOON.

An automatic supply balloon is a gasometer which, when placed in communication with the balloon in the hangar, should keep up in the former an overpressure varying within slight limits, and favorable to the upkeep of the balloon.



Let us suppose this gasometer to be a cylindrical column immersed in a basin filled with water or, better still, with a solution with slight tension of water vapor.

For the sake of simplicity, let us suppose that the walls of the cylinder have the same density as the solution; the difference between the weight of the walls in immersion and that of the solution they displace would be easily made up.

The pressure to be maintained in the lower part of the gasometer may be q, for instance, and about 20 millimeters of water. The difference of level between the interior and the exterior of the gasometer is therefore unimportant.

Let π be the mean plane of the free exterior or interior surface; p being the exterior pressure on that plane, p' the interior pressure. This gives us:

$$p' - p = q$$

now p' = the atmospheric pressure on the lid, that is $p - ha$
+ the weight of the lid per square meter, that is w
+ the weight of the cylindrical wall raised, equally proportionate to the square meter of the base. This weight is naturally proportionate to h; it may be written as hw'
+ the gravity of a column of gas of a height h : hb.

$$p' = p - ha + w + hw' + hb.$$

Therefore

$$p' - p = -ha + w + hw' + hb = \rho.$$

It is desirous that this be equal to q, whatever quantity of gas there may be in the auxiliary, that is, whatever h may be, and especially when $h = 0$. Let h be made to equal 0, $w = q$ will remain. The lid should therefore have a mean weight of q kilos per square meter.

When h is not null, there should moreover be:

$$-ha + hw' + hb = 0.$$

whatever h may be; that is to say

$$w' = a - b = \varphi$$

w' being the quotient of the weight of a height of 1 meter of the cylindrical column by the section of the base of the cylinder.

If p be the weight per square meter of the cylindrical wall, we have:

$$w' = \frac{2\pi rp}{\pi r^2} \qquad w' = \frac{2p}{\gamma} = \varphi.$$

This relation gives p, therefore, for an auxiliary balloon, the diameter of which is chosen according to the capacity selected.

NOTE.—The automatic auxiliary balloon *Juchmés* is similar to the gasometer already examined. The essential difference, beyond the method of its tightness, is that the cylindrical wall of fabric treated with india-rubber not being sufficiently heavy, its weight per square meter of base is brought down to the value $w' = \varphi$ by strings of small ballast bags which rise while the auxiliary is being filled.

The weight of the lid of the auxiliary *Juchmés*, per square meter, should be equal to the required overpressure q.

In this weight of the lid are included, however, the ballast bags of 20 kilos attached higher than the rows of small bags, and which never touch the ground, and any extra ballast bags that may be attached to it.

An auxiliary supply balloon equipped to maintain an average determined pressure at its base may thus be used, if, for instance, the hangar is to occupy a balloon of notably different type, to give a different pressure by modification of the mean weight of its lid; that is, by adding to or reducing the ballast bags of 20 kilos at the top of the strings or on the lid itself.

The friction is really such, on account of the two returns of the pulleys, that the pressure varies by several millimeters more or less than its average value.

Generally speaking, when the auxiliary feeder is connected with the hydrogen producer, the pressure may be, for instance, about 18 or 20 millimeters; if, on the contrary, the feeder empties itself into the balloon or into the compressors, the pressure is nearly 12 or 14 millimeters.

BAROMETRIC LEVELING.

The first application we made of the Principle of Pascal, formulated by equation $p_1 - p_2 = ha$, was to define the variations of pressure when a displacement is made on the vertical from a determined height to the interior of a balloon.

The same principle may also be applied to determine the vertical height or difference of level corresponding to a variation of pressure observed on the barometer; it is this application which constitutes barometric leveling.

It must be remarked that when studying the pressures in balloons, it was granted that the specific gravities of the air and the gas were constant for a vertical displacement of some meters; we have defined these as : average specific gravities. The heights which are the object of calculation in studying leveling are, for instance, altitudes attained by any aircraft or by a sounding balloon. They are of such size that variations in the specific gravity "a" must be taken into consideration along the whole course of the vertical followed.

We shall make use, in estimating a, of a number of approximations successively improved. The closer the approximations, the more complex will be the results obtained.

(A) In the first series of hypotheses we shall suppose the atmospheric air to be deprived of moisture and that its temperature is constant and equal to 0°. Under these conditions, the specific gravity only varies in terms of pressure.

At any moment therefore

$$a = a_1 \frac{p}{p_1}$$

a_1 and p_1 being the specific gravity and the pressure on the starting plane a, and p at any other plane considered.

1. We may even commence by overlooking the variation of specific gravity with the pressure; the air is supposed to preserve, in the

whole area cleared, the same specific gravity as at the level of starting $a=a_1$. The Principle of Pascal will therefore be applied in the form $p_1-p_2=za_1$ when z denotes the variation of altitude.

a_1 is deducted from the gravity of normal air by the proportion

$$a_1=1.293\frac{p_1}{10.333}$$

the pressures being expressed in kgs/m². On carrying out the operations, it comes to about

$$a_1=\frac{p_1}{8,000}$$

or

$$\frac{p_1}{a_1}=8,000.$$

Let us find out what is represented by this number 8,000, equal to the quotient of p_1 and a_1.

p_1 is the pressure at the starting plane, that is, the gravity of the whole vertical column of air actually effective in the atmosphere up to its extreme limit above a base of 1 meter² contained in that plan. a_1 is the gravity of 1 cubic meter of air under that pressure. Their quotient consequently gives the number of cubic meters of air of the same quality, of the same specific gravity, the total gravity of which is equivalent to the whole atmospheric column. If these cubic meters are supposed to be piled up, their number denotes the height of the pile, and we have thus the idea of a column of air of the same quality; that is, homogeneous, the weight of which is equivalent to the atmospheric pressure. This imagery gives us the same impression as the columns of water or of mercury by which we are accustomed to equilibrate and measure the atmospheric pressure.

The height of such a column is called "Homogeneous height."

Its value is constant. No hypothesis has actually been made on the position of the starting plane; we have, moreover, at every moment $\frac{p}{a}=\frac{p_1}{a_1}$, so that if the air were to preserve the same specific gravity a above any plane where the pressure is p, and a were the same specific gravity as in that plane, the height of the atmosphere would be limited to 8,000 meters above.

Returning to the application of the formula of Pascal with the selected hypotheses:

$$p_1-p_2=\frac{zp_1}{8,000}$$

or

$$\frac{p_1-p_2}{p_1}=\frac{z}{8,000}$$

p_1-p_2 being the lowering of pressure or depression. The quotient $\frac{p_1-p_2}{p_1}$ is the relative depression.

The result expressed in the preceding formula is thus stated:

The relative depression is equal to the fraction of homogeneous height cleared.

It is often written thus:

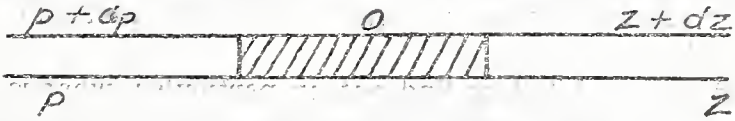
$$\frac{p}{p}=\frac{z}{8{,}000}$$

whence

$$z=8{,}000\frac{p_1-p_2}{p_1}=8{,}000\frac{\triangle p}{p}.$$

2. We shall alter the formula by supposing the section cleared to be filled with air, the specific gravity of which will not be a_1, but $\frac{a_1+a_2}{2}$.

In the first calculation of homogeneous height, the air traversed was always lighter than the air at starting, chosen to represent the quality of the section. In this second hypothesis, the air chosen to represent the quality of the section is lighter than the air traversed at the outset, then heavier than that of the last strata. To a certain extent, there is compensation, which would even be strict if the law of the decrease



of pressures and consequently of specific gravities were a law of proportionality. We shall see further on that such is not the case.

The exactitude obtained here is, however, sufficiently good; the use of the method at an interval of 2,000 meters admits of an approximation greater than 1/100.

The calculation is formulated $p_1-p_2=za$ or a, by the application of the law of Mariotte, and it is of the following value:

$$\frac{1.293}{10.333}\times\frac{p_1+p_2}{2}=\frac{p_1+p_2}{16.000}.$$

Therefore

$$z=16.000\frac{p_1-p_2}{p_1+p_2}\text{ (formula of Babinet).}$$

3. By remaining in the general hypothesis of a constant temperature equal to 0°, the phenomenon of the decrease of pressures is completely analyzed by considering successive and extremely small sections. In such section, as small as may be desired, the specific gravity a may be considered as constant. Let us rise from the height z to the height zdz. The pressure passes from the value p to the value $p+dp$. By remarking that dp is negative, the result is, on applying the formula of Pascal to that extremely small section $-dp=adz$. But

$$a=\frac{p}{8{,}000}.$$

Therefore,

$$8{,}000\,\frac{dp}{p}=dz.$$

We thus get $C-8{,}000\,Lp=z$ where C is a constant quality.

Let C be determined by the starting conditions: We get $C=8{,}000\,Lp$ by calling p_0 the dominating pressure at the plane chosen as the origin of the heights.

Therefore $(Lp_0-Lp)=Z$, when $z=8{,}000\,L(p_0/p)$.

The formula indicates that the law of decrease of pressures with increasing altitudes is a logarithmic law:

When the pressures decrease in geometrical progression, the altitudes increase in arithmetical progression.

The numerical calculation of altitudes is facilitated by the passage to vulgar logarithms. It is easily found that:

$$z=8{,}000\times 2.30259,\ \log.\ (p_0/p)$$

z is taken as $z=18{,}400,\ \log.\ p_0/p$. (Halley's formula.)

The number 18,400 instead of 18,420 is rather too low, but if it is remembered that the number 8,000 adopted as the quotient of 10.333 by 1.293 is itself rather high, it will be granted that 18,400 is very approximate.

Applications.—(a) Let us find out, in particular, what is the altitude, above the level of the sea, of the plane at which the pressure of 10,000 kgs/sq. m. dominates. It is often taken, on account of the facility of its use, as the unit of atmosphere.

Under normal conditions, the pressure dominating the sea level is 10,333 kgs/m^2. The plane of 10,000 is situated at a height of

$$z=18{,}400\ \log.\ \frac{10{,}333}{10{,}000}=261\ \text{m.}$$

above that level.

This is the plane generally taken as the original plane of heights, which authors in some cases call the bottom of the aerostatic ocean.

(b) Let us suppose the pressure to become 1/10 of the initial pressure $p^\circ/p=10$; its logarithm is equal to 1. Therefore $z=18{,}400$. This height is called height at 1/10.

(c) The numerical values of the heights corresponding to different values of p°/p have been calculated by the aid of Halley's formula and set down in the tables of Hergesell (see Marchis': Leçons sur l'Aéronautique; or Espitallier's: La Technique du Ballon). They give the altitudes for the values of p_0/p varying from 1 to 1.25.

For a greater value of p_0/p. the logarithmic law is utilized. For instance, if $p_0/p=1.5$ it might be stated that: $1.5=1.25$ multiplied by 1.2.

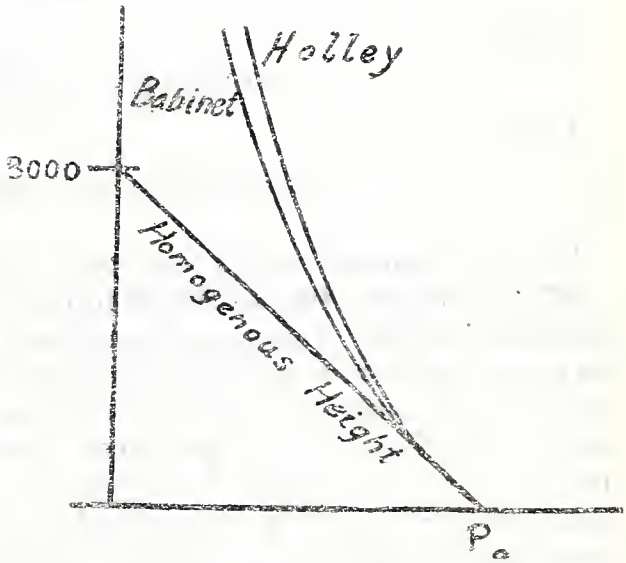
It suffices to add the corresponding heights to the two component factors:

$$1783+1457=3240.$$

Utilizing this process of calculation, the results given will be found assembled in the following table, in some special cases, by applying three formulas of homogenous height, of Babinet and Halley, in which the variable will be the relative depression:

$\frac{P_1-P_2}{P_2}$	Homogeneous height.	$\frac{P_1-P_2}{P_1+P_2}$	Babinet.	$\frac{P_1}{P_2}$	Halley.	Observations.
	Meters.		*Meters.*		*Meters.*	
1/75	105	1/149	106	75/74	106	Absolute depression 1 cm. of mercury.
1/25	320	1/49	326	25/24	326	
1/10	800	1/19	844	10/9	844	Absolute depression 3 cm. of mercury.
1/4	2,060	1/7	2,286	4/3	2,209	4/3=1.333=1.1×1.2121.
1/3	2,666	1/5	3,200	3/2	3,240	3/2=1.5×1.2.
1/2	4,000	1/3	5,333	2	5,540	
3/4	6,000	3/5	9,600	4	11,080	
9/10	7,200	9/11	13,690	10	18,400	

The results may be set down in a diagram. The formula of homogeneous height gives a straight line. The formula of Babinet gives an equilateral hyperbola. The formula of Halley gives a logarithmic or exponential curve. This curve was construeted and utilized by Engineering Capt. Barthés in order to determine the altitude in terms of pressure.



Pressure at high altitudes.—The preeeding table shows that the formula of Halley alone is applicable above 10,000 meters; that of Babinet giving rise to great errors. We shall therefore follow the application of Halley's formula above those altitudes, eonsequently beyond the area which belongs to ballooning. The observation of the logarithmie law shows us that if, at 18,400 meters, the pressure is reduced to $p_0/10$, or 1,000 kgs/m^2 ($p_0=10{,}000$) at 18,400 meters × 2, that is 37 kilometers, the pressure being then $p_0/00$ or 100 kgs/m^2.

At 55.5 km. and 74 km. the pressure beeomes 10, then 1 kg./m^2.

At 55 km. the pressure is no more than 3/4 mm. of mercury, that is, the size of the vaeuum realized in good pneumatic maehines. It is thus seen how mueh the air rarefies, and how rapidly, along a length relatively very small in proportion to the radius of the earth. As regards the end of that atmosphere, its fixation remains

in the realms of scientific hypotheses. Numbers may be given, beyond which the gaseous molecules would detach themselves from the atmosphere, the centrifugal force becoming greater than the Newtonian attraction.

Other numbers may be deduced by observing the trajectories of shooting stars. The numbers, given by different authors, vary from thousands of kilometers to hundreds of thousands of kilometers.

(B) We shall resume the study of leveling by supposing the temperature to be no longer equal to 0°. But we shall still suppose it to be constant, calling the constant value t.

The binomial $(1+\alpha t)$ comes into the specific gravity a as a denominator; elsewhere it comes in as a denominator at Z. Whether a be considered variable or not with the pressure, and whatever may be the mode of variation adopted, the binomial is thus constantly found as a numerator in Z. The three formulas consequently become:

Homogeneous height:

$$z = 8{,}000\ (1+\alpha t)\ \frac{\triangle p}{p}$$

Babinet:

$$z = 16{,}000\ (1+\alpha t)\ \frac{p_1 - p_2}{p_1 + p_2}$$

Halley:

$$z = 18{,}400\ (1+\alpha t)\ \log.\ \frac{p_1}{p_2}$$

The homogeneous height thus expands lineally like the binomial.

(C) The temperature of the atmosphere is not really the same at different heights. Instead of remaining constant, it decreases in an almost continuous manner when we rise; this is explained immediately in the following manner: The gases have very slight absorbing power. They are very pervious to heat, though they retain very little. They do not heat, so to speak, by radiation, but only by conductibility. They are thus the lower strata of the atmosphere which, by contact with the ground, practically assumes the average temperature. The following layers, by contact with the latter and by stirring, are also heated in a rather less degree. The decrease of temperature is marked within the limits of aerostatic areas, up to a height of about 10,000 meters the lowering of temperature amounts to 60° or 80°.

The phenomenon is not regular. Differences in the temperature at different points on the ground, and the variations of the latter are the cause of such irregularity. To these must also be added disturbances caused by the clouds which, formed of small drops of condensed water, absorb the calorific radiancy and heat the masses of air by contact.

This argument enables us to anticipate a limit to the height of the decrease of temperature. In fact, if we suppose local heating to be produced somewhere in the center of the atmosphere, the gaseous mass affected by such heating becomes lighter than surrounding gas, and it rises. It consequently passes to zones where there is less pressure, and expands. Let us suppose such expansion to take place according to the adiabatic law, that is, without exchanging heat with the surrounding gas. The law of this transformation is given by the formula

$$pv = \text{constant}. \tag{1}$$

being the ratio of the two specific heats $y = \frac{C}{c} = 1.141$.

In order to determine the variation of temperature with pressure, for instance, we shall associate the formula of Mariotte-Gay-Lussac with this equation:

$$pv = p_0 v_0 \ (1 + \alpha t)$$

or

$$pv = p_0 v_0 \alpha \left(\frac{1}{\alpha} + t \right) = p_0 v_0 \alpha (273 + t).$$

The simplified statement adopted for this equation is

$$pv = RT. \tag{2}$$

R being a constant $= p_0 v_0 \ \alpha$ and T, which is called absolute temperature, being equal to the centigrade temperature increased by 273.

We shall eliminate v from (1) and (2), for instance, by raising (2) to the power y.

$$p^y v^y = R^y T^y.$$

Dividing factor by factor, we get:

$$p^{y-1} \ T^{y-1} = Cte.$$

If this equation be differentiated:

$$-yp^{y-1} \ T^{-y-1} \ dT + (y-1) \ p^{y-2} \ T^{-y} \ dp = o,$$

or in dividing by:

$$p^{y-2} \ T^{-y-1} = xx.$$

$$ypdT = (y-1) \ T \, dp$$

or

$$\frac{dT}{T} = \frac{y-1}{y} \ \frac{dp}{p}$$

Let us find the variation of pressure and consequently the altitude cleared, corresponding to a variation of temperature of 1°:

$$dT = 1.$$

Let us otherwise adopt $T=273$, which comes to $at=0$. Therefore:

$$\frac{\Delta p}{p}=\frac{y}{y-1}\times\frac{1}{273}=\frac{1.41}{0.41}\times\frac{1}{273}$$

Therefore:

$$z=8{,}000\,\frac{\Delta p}{p}=101 \text{ meters.}$$

The mass of air which has been raised in consequence of local heating therefore cools by about 1° per 100 meters of elevation.

In the adjacent layers, the decrease of the temperature must necessarily be less rapid. In the contrary case, in fact, the mass which rises always remains at a temperature higher than that of the surrounding air, and consequently would continue to rise indefinitely. In the same manner, any local cooling would determine a falling motion in the gaseous mass affected, which would only cease on the ground. The atmosphere would thus be in an unstable state of equilibrium, much disturbed by any variation of temperature. A state of stable atmospheric equilibrium thus entails a more rapid decrease of temperature than 1° every 100 meters.

According to the observations of Flammarion, Col. Renard proposes the adoption of an average law of decrease stating the lowering of temperature to be proportionate to the relative depression. He has admitted the formula to be as follows:

$$t_1-t_2=55\,\frac{p_1-p_2}{p_1}$$

Taking $t_1-t_2=1$, the result is that

$$\frac{p_1-p_2}{p_1}=\frac{1}{55}$$

or

$$z=\frac{8{,}000}{55}=\text{about 150 meters.}$$

According to Capt. Barthés, the decrease would be 1° per 100 meters up to 300 meters above the ground, and of 1° per 200 meters beyond it.

The observations reported by M. Angot show up very exactly the fact that the irregularities due to variations of temperature on the ground may also be extended to a height of 1,000 to 3,000 meters, when the law of decrease can not be correctly asserted. It frequently happens that inversions of temperature are met with; that is, there are instances of the rising of warmer zones over colder ones. This phenomenon is produced, in particular, every night close to the ground, which cools the strata of air by its contact, through radiance and also by conductibility. The effect of these phenomena

of heating on the stability of balloons has been clearly ascertained by Capt. Joux, in the course of numerous night ascents by dirigibles at high altitudes. Beyond 3,000 meters, and up to about 10,000 meters, the phenomenon is more regular and more rapid. It almost attains a value corresponding to adiabatic expansion: About 0.9° per 100 meters. At 10,000 meters, the decrease is brought to a standstill. The temperature would remain practically constant, and would even have a tendency to rise again in what is called the isothermic zone. The thickness of such isothermic zones has not been determined. Its temperature would be −50° to −75°.

To sum up, there is no formula which satisfactorily represents the variation of temperature with altitude. It can only be known by the experience of the moment. Nor can it be measured simultaneously at different altitudes of the zone cleared. Such measurements are usually taken by means of a registering thermometer which registers the temperature of the different air zones in succession.

It is not possible to calculate, analytically, the variable temperature of each air zone by determining its height. A zone in which the temperature varies regularly from t_1 to t_2 is assimilated with a zone in which the temperature would be constant and equal to

$$\frac{t_1+t_2}{2}.$$

If, in consequence, it is supposed that $t=\frac{t_1+t_2}{2}$, the formulas found in paragraph (B) will suit the case.

(D) The endeavor to find greater exactitude leads to consideration of the moisture contained in the air and measured by means of the hygrometer. Let f be the tension of water vapor in the air, p the exterior pressure; the specific gravity of the air is known to be modified, under such conditions, in the ratio $(1-3/8(f/p))$. This factor, entering into a as a numerator, becomes the divisor in z. We shall maintain $(1+3/8(f/p))$, utilizing a familiar approximation, as multiplier in z.

The hygrometric state, like the temperature, varies with the altitude. An average value will be taken as representing the hygrometric state of the zone by supposing, at the same time,

$$\frac{f}{p}=\frac{1}{2}\left(\frac{f_1}{p_1}+\frac{f_2}{p_2}\right).$$

The formula of Halley then becomes:

$$z=18{,}400\ (1+3/8\,f/p)\ [1+\alpha t]\ \log p_1/p_2.$$

Col. Renard's formula.—On account of the uncertain correction of hygrometry, the ordinary air hygrometers being, in fact, rather

inexact instruments, the idea of measuring it may be given up, and only the direction of the correction taken into account by attributing to it an average value.

When it is remembered that moisture generally increases with the temperature, it may be expected, in average cases, that the variation of the specific gravity "a" under the influence of the temperature, is greater than is shown by the coefficient of expansion α. This may be taken into account by adopting a greater value for α; that is, 1/250 or 0.004 instead of 1/273.

The following was taken by Col. Renard:

$$z = 18{,}400\ (1 + 0.004t) \log p_1/p_2.$$

We shall explain t, the average temperature of the section, by adopting Col. Renard's law of decrease:

$$t = \frac{t_1 + t_2}{2}$$

with

$$t_1 - t_2 = 55 \frac{p_1 - p_2}{p_1}.$$

Let us express t, for instance, in terms of the temperature t_2 observed in the balloon (it might otherwise be expressed in terms of t_1 observed on the ground) as follows:

$$t = \frac{t_2}{2} + \frac{t_2}{2}\ 27.5 \left(1 - \frac{p_2}{p_1}\right) \quad t = \frac{t_2}{2} + \frac{t_2}{2}\ 27.5 \left(1 - \frac{p_2}{p_1}\right)$$

$$= t_2 + 27.5 \left(1 - \frac{p_2}{p_1}\right).$$

Therefore:

$$z = 18.400\ (1 + .004t_2 \times .004 \times 27.5 - .004.27.5\ p_2/p_1) \log.\ p_1/p_2$$

or

$$z = 18.400\ (1.11 + .004t_2 - .11\ p_2/p_1) \log.\ p_1/p_2,$$

which formula was long used at Chalais to give the most exactly estimated altitude attained by a balloon.

(E) Finally, the variation of specific weight may be taken into account with the gravity.

Let $g_{\lambda z}$ be the acceleration of weight corresponding to latitude λ and to altitude z; $\frac{g}{0.45}$ to latitude 45° and to the sea level.

It is stated that:

$$g_{\lambda z} = g_{.45}\ (1 - .00259 \cos.\ 2\lambda) \left(1 - \frac{2z}{R}\right).$$

R being the radius of the earth.

Both factors are multiplied in obtaining the specific gravity. We shall therefore maintain as multipliers in z:

$$(1+.00259 \cos. 2\lambda)$$

and

$$(1+2z/R).$$

In conclusion, the most exact formula of barometric leveling, first demonstrated by Laplace and then resumed by M. Angot, is thus expressed:

$$z=18.400\ (1+.00259 \cos. 2\lambda)\ (1+2z/R)(1+f/P)\ (1+\alpha t) \log. p_1/p_2,$$

or practically

$$z=18.400\ [1+.00259 \cos. 2\ \lambda\ 2z/R+f/p+\alpha t] \log. p_1/p_2.$$

It must be observed that the quantity to be determined is in the second factor.

This term of $2z/R$ being small, however, z can be replaced by an approximate value without inconvenience.

Hergesell has constructed tables giving the values of each of the preceding corrections in certain scales of variation of the factors.

It is a recognized fact that with all these improvements the formula can not be employed for heights greater than 1,800 meters. The adoption of an average temperature, in particular, for an interval of such height leads to erroneous results.

In the case of a greater height, it is divided into sections in regular succession; these are less than 1,800 meters thick and are limited, according to the inscriptions of the recording thermometer, by points between which the temperature varies in a consistent manner.

The formula previously stated is applied in each of the sections. The total altitude is equal to the sum of the partial heights thus obtained.

Remarks.—1. The formula of barometric leveling is not only used in aeronautics in calculating the exact height attained by aircraft, particularly for the confirmation of records; it may also be utilized in geodesy together with other topographical processes.

The pressures, in such case, will be measured by means of a mercury barometer instead of an aneroid one; the former is preferable whenever its use is possible. It must be noted, however, that the mercury column should be subjected to the gravity correction.

2. There is another process for determining heights, called hypsometry, based on the following principle: A liquid boils in the open air at the temperature at which its maximum vapor tension is equal to the pressure which it supports.

The tables of vapor tension have been stated by Regnault. The exact measure of the boiling temperature renders it possible to find, in those tables, the value of the external pressure. The barometric record is replaced by a thermometric record. By means of accurate

thermometers the temperature may be known to within 1/50 of a degree. At about 100°, however, the boiling temperature of the water diminishes by 1° with an external pressure of 27 millimeters of mercury; then the diminution of temperature becomes more and more perceptible as the pressure diminishes. The accuracy of the method is fairly good, but its application is not very simple.

THE VERTICAL MOTION OF BALLOONS.

1. THE FREE BALLOON.

We have previously laid emphasis on the variation of the specific gravity of gases in proportion to that of pressure and temperature. A glance at the system of barometric leveling next showed us to what extent the two factors, pressure and temperature, vary in the vertical motion. We shall now discuss the vertical motion of the balloon by following the variation of the forces to which it is subjected.

Let us first consider the case of the free balloon. We shall take no account of the difference of pressure between the balloon gas and the air outside it. It has been seen, in the case of lifting power, that this overpressure lessened the upward vertical force which remained in a slight proportion. The small increase of gravity thus introduced is, moreover, practically constant.

Among the vertical forces acting on the balloon we shall commence by taking no account of purely static forces. We shall consequently overlook the acceleration of motion and the resistance offered by the air to such motion.

The forces are thus reduced to the effect of weight, in a downward direction, and the Archimedean thrust directed upward. The weight effect includes the gravity of the gas and the weight of the solid parts.

Let V be the volume of the gaseous mass, b its specific gravity under the condition of the moment (b is, in consequence, essentially variable); Vb is the gravity of the gaseous mass.

The weight of the solid parts may be designated, for the moment, by a global weight P, which includes the invariable solid weight, and the variable weight of the ballast and of the balloonists. The solid weight is reckoned in air, as is usually understood in the case of the weight of a solid body. It is the true weight, diminished by the thrust of Archimedes. This apparent weight is, in reality, slightly variable when the balloon is displaced in a vertical direction, since the Archimedean thrust varies in proportion to the specific gravity of the air. The differences thus introduced are, however, small in ratio to P. They may certainly be overlooked in that section of height at which the equilibrium of the balloon

must be considered in order to have a determined useful load. Neither shall we take into consideration variations of gravity with varying altitude. The Archimedean thrust on the gaseous mass is equal to Va.

These forces are constituted by subtraction. The resultant, which is in an upward direction, is equal to $Va - Vb - P$.

If such resultant is null, there is equilibrium; if not, it is called loss of balance. If the expression thus estimated is negative, the subtraction being made in the direction indicated, it is evident that the resistance is in a downward direction.

Let us therefore suppose that—

$$R = Va - Vb - P.$$

Inflated and slack balloons.—A distinction must be made between the conditions of the balloon when it is completely or partially inflated. In the former case the envelope is tight and the volume is constant and equivalent to its maximum. But the gravity of the gas contained diminishes gradually on account of the quantity that escapes through the sleeve of the appendix, though the balloon is said to be fully inflated.

In the second case the envelope makes folds, so that the volume is less than the total capacity of the envelope and is variable. The weight, on the contrary, is constant, since the gaseous mass can expand without exhaustion through the appendix. The balloon is said to be slack.

INFLATED BALLOON OR CONSTANT VOLUME.

Let us first consider the case of the fully inflated balloon, or the balloon with constant volume.

So long as a balloon is full it does not descend. A descending motion—that is, the arrival of the balloon in a higher zone of pressure—is, in fact, accompanied by a contraction of the gaseous mass, so that the balloon becomes slack. A balloon remaining fully charged can not, therefore, be subjected to negative losses of balance.

Take the case of such a balloon on the ground, at the moment of weighing, that is, in a state of equilibrium—

$$Va_1 - Vb_1 - P_1 = 0.$$

We shall take it for granted that the gas is subject to the same pressure and temperature as the outer air, which is sufficiently exact in the case of the first general results about to be stated.

Supposing that

$$\varphi_1 = a_1 - b_1.$$

It may be stated that

$$V\varphi_1 - P_1 = 0. \tag{1}$$

Let us produce a positive loss of balance. For instance, a certain quantity of ballast might be dropped, such as l_1 or unballasting. The positive loss of balance is equal to l_1.

Under the action of this loss of balance the balloon ascends. At the same time the lifting power φ lowers with the external pressure. The loss of balance $V\varphi - P$ consequently decreases and finally disappears. This takes place on a plane where the lifting power has the value of φ_2, so that

$$V\varphi_2 - (P_1 - l_1) = 0. \tag{2}$$

From a static viewpoint the balloon will be equilibriated on that plane.

Let another quantity of ballast, l_2, be dropped. The balloon climbs. When there is a new value φ_3 of the lifting power, the loss of balance is again canceled—

$$V\varphi_3 - (P_1 - l_1 - l_2) = 0. \tag{3}$$

The operation may be repeated. It is seen that the equation of equilibrium, after n droppings of ballast, comprises a solid weight $(P_1 - l_1 - l_2 \ldots l_n)$, which evidently does not depend on the order of the operations. The value of φ is consequently given by the equation, and for that reason the altitude attained depends only upon the total quantity of ballast dropped and in no wise upon the manner in which it has been dropped.

First law.—By comparing equations of equilibrium on two successive planes another law may be deduced. For instance, equations (1) and (2), previously stated, characterize the equilibrium on planes 1 and 2. We shall now subtract those equations factor by factor.

The result is—

$$V\varphi_1 - V\varphi_2 = l_1. \tag{4}$$

Now, $V\varphi_1$ is the total lifting power of the mass of gas contained in the balloon on plane 1, $V\varphi_2$ being the total lifting power of the gaseous mass remaining in the balloon on plane 2.

The total lifting power of the gaseous mass originally contained in the balloon is equal to that of the remaining gas plus that of the gas which has escaped, since we know that the total lifting power of an invariable gaseous mass, subjected at every instant to the pressure and temperature of the outer air, is constant.

The first factor of equation (4) therefore represents the total lifting power of the escaped gas when passing from plane 1 to plane 2.

Second law.—When a balloon climbs, it ejects a gaseous mass, the total lifting power of which is equal to the original unballasting.

Unballasting formula.—It is now important that we should know the height attained after a certain amount of unballasting, which problem will present itself in practice as well as in theory. How much ballast is it necessary to drop in order to reach a stated height?

This question entails the study of a series of more and more complicated special cases, in the same manner as in studying barometric leveling.

(A) Suppose the temperature of the air and that of the gas to be constant and equal to one another; let t be their value, whether it differ from 0° or otherwise.

This hypothesis is more restrictive than that made up to the present, and according to which the temperatures of the air and of the gas alone were equal. The preceding formulas therefore hold good.

Let us resume formula (4) and replace φ_2 in terms of φ_1. The law of Mariotte alone controls the transfer from one value to the other:

$$\varphi_2 = \varphi_1 \; p_2/p_1$$

Therefore—

$$V\varphi_1 \; (1 - p_2/p_1) = l,$$

or

$$\frac{1}{V\varphi_1} = \frac{p_1 - p_2}{p_1}$$

The first factor is the ratio of the unballasting to the total lifting power. Col. Charles Renard called it the relative unballasting and expressed the following law:

The relative unballasting is equal to the relative depression.

The depression being known, we are faced with a problem of barometric leveling in order to find the height.

Let us first take the formula of homogeneous height and see the result of its application:

$$z = 8000 \; (1 + \alpha t) \frac{p_1 - p_2}{p_1}$$

Here we have—

$$z = 8000 \; (1 + \alpha t) \frac{1}{V\varphi_1} \qquad (5)$$

If, in particular, the constant temperature is supposed to be equal to zero, the formula stands thus:

$$z = 8000 \frac{1}{V\varphi_1} \qquad (5')$$

By the interpretation of this formula we can enunciate a certain number of laws:

Third law.—1. Supposing z to be determined, it is found that the loss of balance should be proportionate to the volume of the balloon and to the lifting power of the gas which inflates it in order to attain a given height.

2. If we suppose 1 to be determined, z varies in inverse ratio to V_1.

Fourth law.—*a.* For a given unballasting of two balloons inflated with the same gas, it is therefore the smaller one which climbs the higher.

Fifth law.—*b.* For a given unballasting of two balloons of the same volume, the one which is inflated with the heavier gas will climb higher than the other one.

These two propositions combined, *a* and *b*, are often called an aerostatic paradox.

c. The lifting power φ_1 does not depend only upon the nature of the gas. We know that it varies in proportion to the pressure. If φ_0 is its value at 0° and with a pressure of 10,000, at 0° and with a pressure p_1 its value will be equal to

$$\varphi_0 \frac{p_1}{10{,}000}$$

φ_1 will therefore be so much the smaller when p_1 is smaller; that is, when the balloon is at a great height. Therefore:

Sixth law.—In the case of a given unballasting, a balloon climbs all the higher when its unballasting takes place at a higher altitude.

d. The lifting power varies equally with the temperature, in the proportion stated by the law of Gay-Lussac.

$$\varphi_t = \varphi_0 \frac{1}{1+\alpha t}$$

Let us take (5) into the formula instead of (5′), since the homogeneous height is itself multiplied by the binomial

$$z = 8000\ (1+\alpha t)^2\ l/V\varphi_0$$

By overlooking the term of α_2 it might be stated thus:

$$z = 8000\ (1+\alpha t)\ l/V\varphi_0.$$

For a double reason, the value of z is therefore increased by a rise in temperature. The law may be expressed as follows:

Seventh law.—A balloon climbs higher in summer than in winter when acted upon by a determined unballasting.

Application of the formulas of Babinet and Halley.—The directions given in the formula of homogeneous height are often sufficient in practice. The laws thus obtained remain true, if a more exact for-

mula be made use of, in determining the height attained. We shall examine the alteration brought to bear on the estimation of z by employing the formulas of Babinet and Halley.

Let us suppose, for instance, that the constant temperature be 0:

Babinet.—

$$z = 16{,}000\,\frac{p_1 - p_2}{p_1 - p_2} = 16{,}000\,\frac{1 - \frac{p_2}{p_1}}{+}$$

But

$$1 - \frac{p_2}{p_1} = \frac{l}{V\varphi_1}$$

Whence we deduce

$$1 + \frac{p_2}{p_1} = 2 - \frac{l}{V\varphi}$$

Therefore

$$z = 16{,}000\,\frac{\frac{l}{V}}{2 - \frac{l}{V\varphi}} = 16{,}000\,\frac{1}{2V\varphi - l}$$

or

$$z = 8000\,\frac{1}{V\varphi - \frac{l}{2}} \tag{6}$$

Halley.—A calculation in logarithms is needed to obtain the result numerically. A large number of results have been recorded for balloons of different capacity inflated with hydrogen or illuminating gas in the unballasting tables drawn up by Hergesell.

Another approximate formula.—If, in Halley's formula, the logarithm be developed in series, and if we stop at the first term, a fractional expression is obtained, which may be calculated by the ordinary process.

This comes to—

$$z = 8000\,\frac{l}{V\varphi = l} \tag{7}$$

It is not worth while to make the calculation of this formula; it shows no improvement on the simple formula of homogeneous height, as will be seen by the following comparison.

Suppose we have a normal balloon, of which the total lifting power is 600 kgs. on the plane of 10,000. The three formulas of unballasting give the following results:

Unballasting, L	Homogeneous height, $8000\frac{L}{600}$	Babinet, $8000\frac{L}{600} - \frac{1}{2}$	Halley, $8000\frac{L}{600} - L$	Tables of Hergesell.
30k	400	410	420	410
100k	1,333	1,454	1,600	1,460

From this a good approximation may be inferred in employing the formula of Babinet.

Coefficients of stability and of mobility.—The preceding study is evidently imperfect, on account of the hypothesis which has been made on the subject of temperature. The results obtained may, however, serve as an index in average cases and it is a good plan to keep a certain number of them in mind.

For instance, the "coefficient of mobility" of a balloon is the name given to the number of meters climbed by it after 1 kilo of ballast has been dropped. An average number is adopted, corresponding to the temperature of 0 and to the value of the lifting power on the plane of 10000.

Under these conditions the following number is obtained for the normal balloon:

$$m = 8000 \frac{1}{600} = 13 \text{ m. } 33 \text{ cms.}$$

13.50 m. would be adopted.

For the siege balloon, the mobility coefficient adopted is of the value of 9.70 m.

A magnitude contrary to the preceding is defined and called the "coefficient of stability." It is the weight of ballast that must be thrown to raise the balloon 1 meter, or it may be the diminution undergone by its total lifting power for an elevation of 1 meter. Its value s for the normal balloon, for instance, is such that

$$1 = 8000\ s/600.$$

We may therefore have

$$s = \frac{1}{m} = .074 \text{ k.}$$

The stability coefficient adopted for the siege balloon is .103 kgs.

(B) *In case of variable temperature.*—We shall suppose, moreover, the temperature of the gas and that of the air to be different, each of them varying in some way. We shall take the most general case, with the object of shortening the calculations. The following particulars apply to it: First, when the temperature is variable, but remains the same for the gas and for the outer air. Second, when the temperature is different from the gas and the air, but when such difference is constant.

In that case, the simple law of Charles Renard can no longer be applied (relative unballasting = relative depression). The unballasting is connected with the depression by the action of the temperature. It is this action which we are about to establish.

On plane (1), let p_1 be the pressure, t_1 the temperature of the air, t'_1 that of the gas, a_1 and b_1 the specific gravities of the air and of the gas, P_1 the total lifting power.

We shall suppose that there is equilibrium.

Therefore:

$$V(a_1 - b_1) = P_1.$$

Let a quantity L be unballasted. The solid weight becomes $P_2 = P_1 - L$.

The balloon recovers its equilibrium in a zone 2 where the preceding components have the following values:

$$p_2 t_2 t'_2 a_2 b_2.$$

The equilibrium ratio is thus expressed

$$V(a_2 - b_2) = P_1 - L_2.$$

We shall divide, factor by factor, the two equations of equilibrium. The result is as follows:

$$\frac{V(a_2 - b_2)}{V(a_1 - b_1)} = \frac{P_1 - L.}{P_1}$$

or

$$\frac{a_2 - b_2}{a_1 - b_1} = 1 - \frac{L}{P_1} \tag{8}$$

But we may express a_2 and b_2 directly in terms of a_1 and b_1.

$$a_2 = a_1 \frac{p_2}{p_1} \frac{1 + \alpha t_1}{1 + \alpha t_2}$$

or, by using an artifice of calculation several times employed:

$$a_2 = a_1 \frac{p_2}{p_1} [1 + \alpha (t_1 - t_2)]$$

in the same way:

$$b_2 = b_1 \frac{p_2}{p_1} [1 + \alpha (t'_1 - t'_2)]$$

Let us suppose that:

$t_1 - t_2 = \epsilon$ lowering of the temperature of the air;

$t'_1 - t'_2 - \epsilon' =$ lowering of the temperature of the gas.

By taking $\frac{p_2}{p_1}$ as a factor, it may be stated thus:

$$a_2 - b_2 = \frac{p_2}{p_1} [a_1 - b_1 + \phi c \epsilon a_1 - b_1 \alpha \epsilon']$$

Let us introduce as a factor $a_1 - b_1$ to meet the form required by the formula

$$a_2 - b_2 = \frac{p_2}{p_1} (a_1 - b_1) \left[1 + \frac{\alpha}{a_1 - b_1} (a_1 \epsilon - b_1 \epsilon') \right] \tag{8'}$$

From this we deduce

$$\frac{a_2-b_2}{a_1-b_1} \text{ or } 1-\frac{L}{p_1}=\frac{p_2}{p_1}\left[1+\frac{\alpha}{a_1-b_1}(a_1\epsilon-b_1\epsilon')\right]$$

whence, by remarking that the coefficient of p_2/p_1 consists of 1 plus a smaller term,

$$\frac{p_2}{p_1}=\left(1-\frac{1}{p_1}\right)\left[1-\frac{\alpha}{a_1-b_1}(a_1\epsilon-b_1\epsilon')\right]$$

$$\frac{p_1-p_2}{p_1}=1-\frac{p_2}{p_1}=\frac{L}{p_1}\left(1-\frac{L}{p_1}\right)\frac{\alpha}{a_1-b_1}(a_1\epsilon-b_1\epsilon')$$

Such is the relation between the depression and the unballasting.

Let us carry the value thus found into the formula of homogeneous height:

$$z=8{,}000(1+\alpha t)\frac{p_1-p_2}{p_1}.$$

It must be noted that the temperature t entering this formula is an average temperature $t=\frac{t_1+t_2}{2}$.

We thus find, for the height resulting from unballasting 2,

$$z=8{,}000(1+\alpha t)\left[1/p_1(1-1/p_1)\frac{\alpha}{a_1-b_1}(a_1\epsilon-b_1\epsilon')\right] \tag{9}$$

or

$$\underline{z=8{,}000(1+\alpha t)p_1}+8{,}000\,(1+\alpha t)\,(1-1/p_1)\frac{\alpha}{a_1-b_1}(a_1\epsilon-b_1\epsilon').$$

The expression z therefore consists of a principal and underlined term which is none other than the value previously found by the application of the formula of homogeneous height in the case when the temperature was supposed to be constant (formula 5). A corrective term is added to that principal term.

In calculating the corrective term, which is evidently small, or when α is a factor, we are authorized to make approximations. 1 may thus coincide with the product of the two factors $(1+\alpha t)$ and $(1-t/p)$ each very near to 1, and one of which is generally greater than 1 and the other less. The corrective term is then expressed as follows:

$$8{,}000\frac{\alpha}{a_1-b_1}(a_1\epsilon-b_1\epsilon').$$

1. Let us first take a special case $\tau=\tau'$.

It must be remembered that this expresses—

$$t_1-t_2=t'_1-t'_2.$$

It may be also stated that—

$$t'_2 - t_2 = t'_1 - t_1,$$

or

$$\theta_2 = \theta_1.$$

The quantity θ has already been defined as the difference between the temperature of the gas and that of the air, and it has been called heating. The particular case in question supposes that the heating does not vary. There is then all the more reason, supposing the heating to be nonexistent, for there to be no difference between the gas and the outer air.

The corrective term becomes—

$$8000\frac{\alpha}{a_1 - b_1}(a_1 - b_1)\tau = 8000\alpha\tau.$$

(*a*) The cooling τ is local. It is produced in the zone of equilibrium of the balloon, without any initial unballasting. The principal term of z is therefore nonexistent. The alteration of the zone of equilibrium is indicated merely by the corrective term, which is $8{,}000\alpha$, that is, 29 meters per degree of the simultaneous cooling of the surrounding air and the gas.

Eighth law.—Marchis has enunciated the following law: The height of the zone of equilibrium of an inflated balloon varies by about plus or minus 30 meters every time the temperature of the air diminishes or increases by 1°, whatever may be the size of the balloon, its weight, the nature of the gas that inflates it, or the height at which it is found.

(*b*) The lowering of temperature τ is distributed over a zone that is cleared under the impetus of an initial unballasting 2. Under these conditions the principal term and the corrective term intervene at the same time to modify z.

The principal term is expressed as follows:

$$8{,}000(1 + \alpha t)t/p_1.$$

But

$$t = \frac{t_1 + t_2}{2} = t_1 - \frac{t_1 - t_2}{2} = t_1 - \frac{\tau}{2}$$

the principal term may therefore be expressed thus:

$$= 8000\left[1 + \alpha\left(t_1 - \frac{\tau}{2}\right)\right]t/p_1.$$

It therefore diminishes by $\frac{\alpha}{2}$, that is, about 1/550, per degree of lowering of the temperature in a vertical direction, that is to say, when $t = 1$. Granting that the lowering of temperature by 1° be

obtained by clearing a height of 160 meters, which average number has been previously indicated, the principal term is seen to be diminished by a value of about $\frac{1}{3}$ of a meter, which is negligible. At the same time, the corrective term has attained the value of $8{,}000\alpha$, that is, 29 meters. Its importance is far more accentuated by the slight preceding error. The zone of equilibrium, which would be situated, according to the unballasting decided upon, at a height of 160 meters, in an atmosphere where the temperature would be constant and equal to its value on the ground, is brought to about 190 meters, in consequence of the decrease of temperature.

Granted the constancy of the phenomenon of variation of temperature on the vertical, the elevation of the zone of equilibrium is always in the same ratio. The following statement may be made:

Ninth law.—Granting the decrease of temperature to be regular, that is, 1° every 160 meters, the height of equilibrium calculated in accordance with the unballasting decided upon and with the constancy of the temperature is raised in the ratio of 19/16.

2. In general

$$\tau = \tau'.$$

Let it be remarked that

$$\tau = \tau' - \theta_2 = \theta_1 = \Delta\theta.$$

The corrective term is thus expressed, by subtracting and by adding,

$$8{,}000\frac{\alpha}{a_1 - b_1}(a_1\tau - b_1\tau + b_1\tau - b\tau');$$

or

$$8{,}000\frac{\alpha}{a_1 - b_1}[(a_1 - b_1)\tau + b_1(\tau - \tau')$$

or

$$8{,}000\alpha\tau + 8{,}000\frac{\alpha b_1}{a_1 - b_1}\Delta\theta.$$

To the effect τ which we have just studied, an effect $\Delta\theta$ is added. The greater is the specific gravity of the inflating gas, the greater is the latter effect. Let us compare, by way of example, illuminating gas whose lifting power would be 700 grammes with hydrogen gas of 1,100 grammes lifting power, on the plane of 10,000 ($\alpha_1 = 1{,}250$).

$$\text{Illuminating gas} \left\{ \begin{array}{l} \dfrac{b_1}{a_1 - b_1} = \dfrac{550}{700} \\[2ex] 8{,}000 \times \dfrac{b_1}{a_1 - b_1} = 25 \text{ meters.} \end{array} \right.$$

$$\text{Hydrogen} \ldots\ldots \left\{ \begin{array}{l} \dfrac{b_1}{a_1 - b_1} = \dfrac{150}{1100} \\[2ex] 8{,}000\dfrac{\alpha b_1}{a_1 - b_1} = 4 \text{ meters.} \end{array} \right.$$

Tenth law.—When the excess of the temperature of the inflating gas over that of the outer air (heating) increases by 1°, the zone of equilibrium rises by about 4 meters for a balloon inflated with hydrogen, and by about 25 meters in the case of a balloon inflated with lighting gas.

NOTE.—The two effects τ and θ are added when, in particular, a balloon enters a cold zone before there has been time for an exchange of temperature between the interior and the exterior. The special stability of a balloon in the upper part of a sea of clouds directly heated by the sun is a consequence of this double phenomenon; it is the fact that the balloon finds it equally difficult to rise above it as to plunge into it again.

The steadiness of the balloon in nocturnal ascents is a consequence of the same phenomenon; we know, as a matter of fact, that the decrease of temperature is then reversed in the vicinity of the ground. Such distribution of temperature therefore tends to raise a balloon which is sinking, and to drive back a climbing balloon.

SLACK BALLOONS OR BALLOONS WITH CONSTANT GRAVITY.

2. The gaseous mass enclosed in a balloon of this description does not completely fill the capacity of the balloon. The restricted volume which it occupies is limited to the lower part by a fabric which makes folds and is slightly raised on account of a slight excess of exterior pressure, instead of having an unbroken outline corresponding to its maximum capacity. The gaseous mass may therefore dilate freely, without exhaustion by means of the appendix. The quantity of gas contained in the interior is thus variable, while the balloon is slack; and the total lifting power of a constant mass of gas is in itself constant. This quality, which forms the basis of the theory of the slack balloon, is worthy of being recalled here. It supposes the pressure of the gas to be the same at every moment as the external pressure, and it is true in the case of variable temperatures, provided the temperature of the gas and the temperature of the air be likewise equal at every instant. We shall therefore let the pressure and the temperature vary at the same time, in the gas and in the air.

Take the case of a zone of navigation in which the pressure is p_1 and the temperature t_1; a_1 and b_1 are the specific gravities of the air and of the gas; and under such conditions, v_1 (round) is the volume of the constant gaseous mass.

On a new plane, the same factors have the following values:

$$p_2\ t_2\ a_2\ b_2\ v_2.$$

By hypothesis

$$v_2\ b_2 = v_1\ b_1.$$

But if the gaseous mass is constant, its volume varies according to the laws of Mariotte and of Gay-Lussac:

$$v_2 = v_1 \frac{p_1}{p_2} \frac{1+\alpha t_2}{1+\alpha t_1}.$$

As for the specific gravity of the air, the variations it undergoes under the action of the factors pressure and temperature are the contrary of the preceding, and are as follows:

$$a_2 = a_1 \frac{p_2}{p_1} \frac{1+\alpha t_1}{1+\alpha t_2}.$$

Therefore:—

$$v_2\, a_2 = v_1\, a_1.$$

The Archimedean thrust exerted on the balloon is therefore invariable. In summing up, it may be said that while the solid weight itself is stationary, the loss of balance applied to the balloon is constant:—

$$v_2\,(a_2 - b_2) - P = v_1\,(a_1 - b_1) - P.$$

In this course of reasoning, no hypothesis whatever is made on the relative values of p_1 and p_2. The quality is true, whatever may be the direction of the motion produced. A slack balloon, whether climbing or descending under the impetus of a loss of initial balance, is always subjected to the same loss of balance. Consequently:

1. If the balloon descends, it only stops on reaching the ground.

2. If the balloon climbs, it attains its zone of plenitude, where it behaves like a fully inflated balloon subjected to the same unballasting as that of starting. It climbs above its zone of plenitude to a distance corresponding to the quantity of the unballasting.

The slacker the balloon at the moment of starting, the higher will be the zone of plenitude. This proposition and those preceding it are combined in a single enunciation, apparently paradoxical, in the Rules for Military Ballooning, first volume. It runs as follows:

In the case of a given unballasting, a slack balloon rises so much higher when it is less inflated at starting; when the total volume is smaller and the aerostatic gas stronger.

The zone of equilibrium therefore depends, on the whole, on the remaining solid weight alone. It is consequently the same for a balloon which starts fully inflated or for a balloon starting slack under the same conditions of solid weight. It suffices that the slack balloon can actually leave the ground. If it is too slack, that is, if its zone of plenitude is higher than the zone of equilibrium corresponding to its total solid weight, it will not leave the ground.

THE DROPPING OF BALLAST AND ACTION OF THE VALVE.

A motion started in one direction will thus continue within the limits stated unless there is intervention on the part of the balloonist. He disposes of the ballast to check a downward motion. He should naturally throw more ballast than is necessary to cancel the loss of negative balance which would make the balloon descend. The latter being simply annulled, the balloon would continue to descend by virtue of its acquired speed. The direction of such speed must be changed, and the balloon will at once begin to climb again.

It would naturally climb higher than the preceding zone of equilibrium.

From this, it results that the zones of equilibrium of an inflated balloon rise in progressive proportion in the course of navigation. To check a climbing motion, the pilot has control of the valve. Through the action of the valve, the balloon assumes additional weight equal in quantity to the lifting power of the escaped gas. Hence the necessity, in maneuvering, to take into account the volume of escaped gas. The rules thus define the type of valve action "coup de soupape"—the increase of weight which it produces is of the same value as the regulation ballast dropping for the same balloon.

If the balloon continues to climb in spite of the action of the valve, in the course of a maneuver, it will not yet be full at its preceding zone of equilibrium. It will have to rise above that zone in order to be full, and still higher to annul the unballasting which made it climb. This higher elevation of the zone of equilibrium leaves the balloon at such an altitude at the end of the maneuver, that it would be necessary to provide a far greater reserve quantity of ballast in order to redescend. This leads us to the conclusion that the working of the valve necessitates a far greater consumption of ballast.

The preceding study is diagrammatical on account of its not taking into consideration accidental variations which might occur in the factors of equilibrium. The variations of exterior or interior temperature will have particular influence on the motions of the balloon in the direction already pointed out with regard to the fully inflated balloon.

1. The lowering of the external temperature tends to make the balloon rise, or to check its descending motion.

2. The elevation of the internal temperature produces the same effect.

OVERHEATING.

Variations of the inner temperature are systematically produced in the descending or climbing motions of the balloon by reason of adiabatic compression or expansion.

In a slack balloon which descends, for instance, the increase of temperature is 1° per 100 meters of descent. But the outer temperature increases only by about 1° per 160 meters. A term of overheating thus arises, which assumes more and more importance. If the regularity of the phenomenon be admitted, the variation of the total lifting power per meter cleared on the vertical may be determined, or the coefficient of stability. Marchis states the coefficient of stability $s' = .13s$ for a balloon inflated with hydrogen, that being the stability coefficient defined for the same balloon when fully inflated. Such coefficient of stability is not only slight, but is based on a fragile hypothesis, so that a coefficient of stability can not be reckoned upon in the case of the slack balloon.

If a slack balloon is at equilibrium at any moment, it must be recognized that such equilibrium is unstable. Experiments show that if the equilibrium is interrupted the motion generally continues up to the limit of the section in which the balloon is slack; and this confirms the statement made at the commencement in the simple hypothesis which takes no account of differences of temperature.

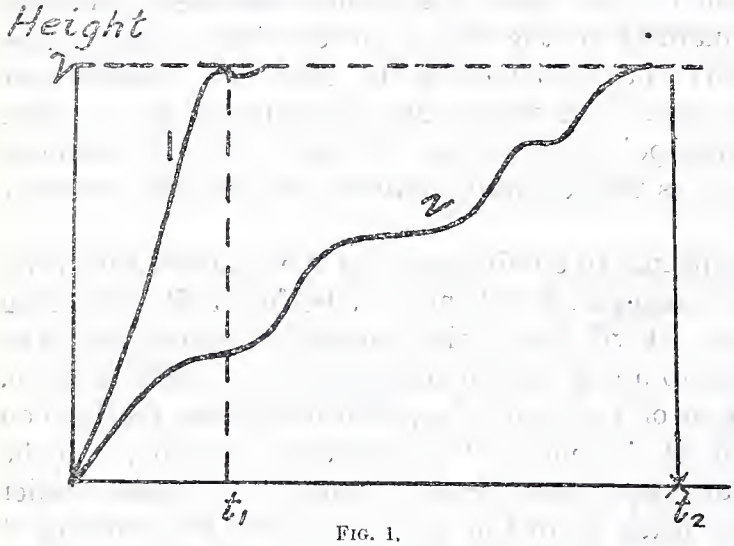


Fig. 1.

Note.—It follows from the preceding propositions that for an ascent of long duration, it is advisable to start with an entirely full balloon and to carry the largest quantity of ballast.

As a matter of fact, while the slack balloon rises directly to its zone of plenitude, as in diagram 1, for instance, the full balloon will reach the same zone by stopping at successive zones of equilibrium determined by the sparing use of all the ballast carried, according to a diagram which may be represented by 2. From the moment when the two balloons are at equilibrium at a height z, they may continue their ascent under identical conditions. But the balloon will have gained an interval of time represented by $t_2 - t_1$ (Fig. 1).

DIAGRAMMATICAL CONSTRUCTION.

The results obtained in studying the fully inflated balloon may be diagrammatically shown as in the case of the slack balloon. Such a diagrammatical scheme has even been utilized by Marchis in establishing his laws. (See pages 62 and 63, Fig. 1 and Fig. 2.)

Let the gravities be expressed in abscissæ and the heights in ordinates. The gravity of the volume of air displaced by the full balloon or Archimedean thrust Va is represented by a curve such as 1, supposing the specific gravity to vary according to the law of Halley. The gravity of the gas Vb is represented by a curve 2, the solid weight P is represented by a parallel to one of the ordinates taken at P. By taking back the gravity of the ga Vb, to that straigh line taken as the axis of the ordinates, curve 3 represents the total weight P Vb, or the effect of weight on the balloon (fig. 2). The balloon will be at equilibrium when $Va = P + Vb$, that is, at the height corresponding to the meeting point of curves 1 and 3.

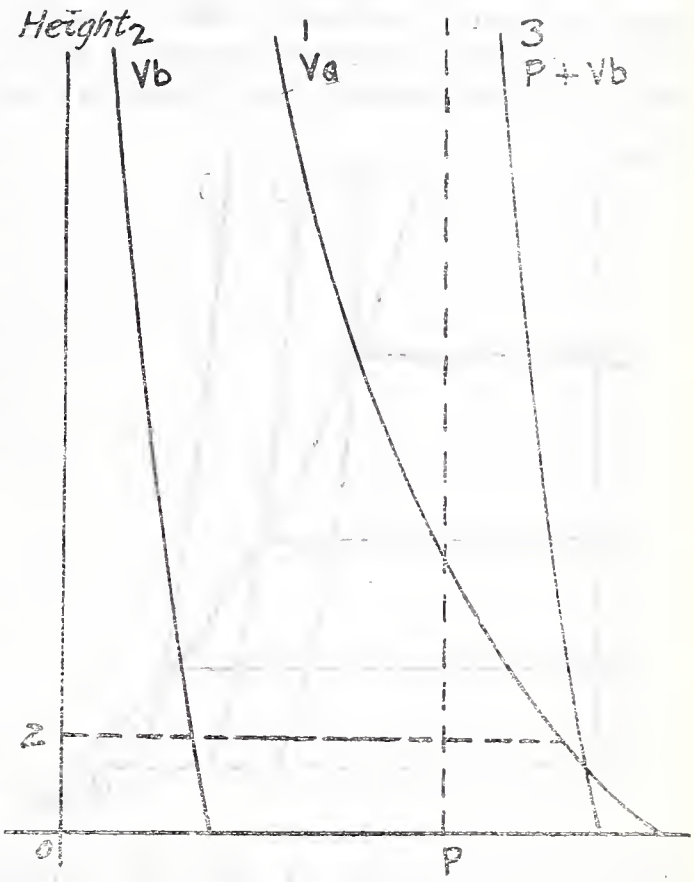


FIG. 2.

If curve 3 be displaced toward the left by equal stages, corresponding to the dropping of successive and equal quantities of ballast, the heights cleared in consequence of such unballasting will progressively be $BC > AB$ (fig. 3).

The propositions of the aerostatic paradox would likewise be encountered by remarking that the smaller is the volume of the balloon and the heavier the aerostatic gas, the smaller is the angle made by the two curves Va and Vb. In the case of a given lateral displacement, their bisecting point is therefore pushed higher up.

The qualities of the slack balloon would be expressed in a similar manner in a diagram of the same kind.

APPLICATION OF THE LAWS OF THE VERTICAL MOTIONS OF BALLOONS TO DIRIGIBLE AIRSHIPS.

From the viewpoint of vertical motion the dirigible is distinguished from the free balloon by the introduction of a vertical force called dynamic among the factors of equilibrium. This force, caused by the action of the air on the planes and on the streamline, may be directed in one direction or the other, downward or upward, and regulated in dimension to a certain extent by the maneuver of the altitude wheel control. The pilot, having at his disposal a new instrument, has therefore more control over his navigation. If such instrument be sufficiently powerful, he can choose at will, so to speak, his zone of navigation below a certain upper limit. He no longer need have constant recourse to static means, particularly ballast. While free ascents are limited in duration by the expenditure of ballast, it is not generally the case in ascents by dirigible. Consumption of gasoline, strain or accidents in the working of the mechanical parts, or fatigue on the part of the crew are the causes that usually set a limit to attempts at ascents of long duration.

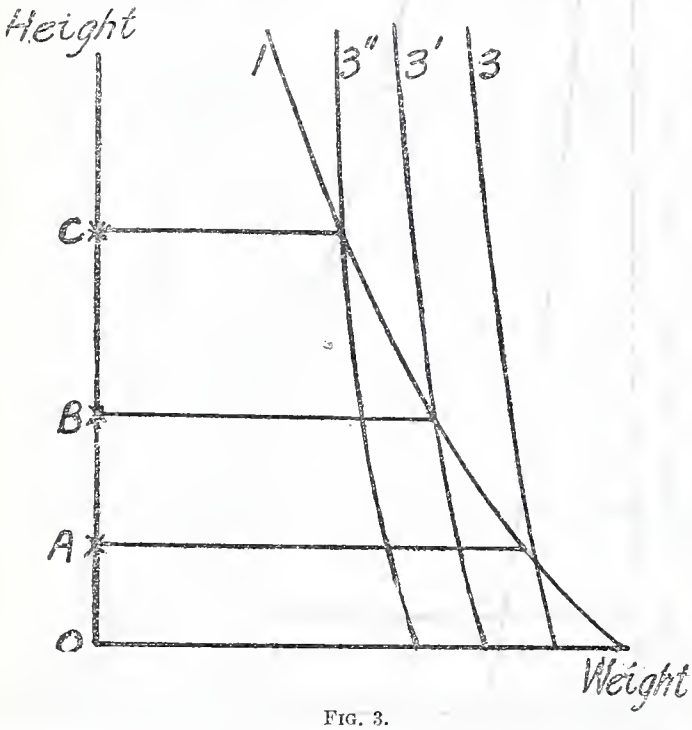


Fig. 3.

Having considered the air resistance, we resume the subject of dynamic force, its extent and its application. Its particular importance must be noted in the case of rigid balloons, on account of their high speed and their elongated prismatic shape. Such dynamic means have sometimes been made use of in making rigid balloons climb appreciably higher than their static zone of equilibrium. In those cases the weight of the balloon is equal to the lifting power of the gas ejected above its zone of equilibrium. In descending again, it shows the quality of the slack balloon—that is, it maintains the same loss of balance. It is therefore heavy on landing and so much the heavier when it is raised higher on its planes. A number of the Zeppelin catastrophies have been due to that cause. At present, such

sources of accident are reduced by lessening the time during which the balloon is deprived of its dynamic resources. With that object, two propelling groups out of the five with which the balloon is generally provided have a reverse movement. The balloon can thus reach the landing ground at a very low altitude, and at a speed which permits the pilot to maneuver and to stop almost instantaneously by means of his reverse-movement propellers.

In flexible balloons working at high speed the same error may be committed. It should be absolutely avoided. The installation of backward movement propellers is not, however, generally adopted by French dirigible constructors.

The characteristic of the flexible balloon, in contradistinction to the rigid balloon, is that of having the permanence of its outer form insured by one or several ballonets, in which air blowers inject air under pressure. The air injected during descent may occupy, with the inflating gas, the same volume as that occupied by the gas ejected during the corresponding climb. The contractions entailed by diminutions in the temperature of the gas lead to a similar need for the introduction of air into the ballonets.

We shall first determine the volume of air which must be injected in descending from a determined height. The temperature will be supposed to be constant.

In the case of a small height being cleared, where the variation of pressure $\triangle p$ is small, we know that the variation of volume $\triangle V$ is given by

$$\frac{\triangle V}{V} = \frac{\triangle p}{p} \text{ (absolute values).} \tag{1}$$

At the same time

$$\frac{\triangle p}{p} = \frac{3}{8{,}000}$$

therefore

$$\frac{\triangle V}{V} = \frac{3}{8{,}000}.$$

The *Fleurus*, in descending 400 meters, will thus have in its ballonets an approximate volume of $6{,}500 \times 400/8{,}000 = 325$ m3 of air.

Let us now suppose a greater height to be cleared, and consider in particular the greatest height which the balloon can attain. The volume of air blown into the ballonets during descent will show what should be the total volume of the ballonets.

Let p be the pressure on the ground and p' the pressure at the highest plane. If V be the volume of the balloon, at plane p' the gaseous mass which fills the balloon at starting occupies a volume V', so that

$$p' \; V' = p \; V.$$

The volume of the gas exhausted, measured under that pressure p', is therefore $V'-V$. Now it is evident that the quantity of the gaseous mass ejected during the climbing movement would be just sufficient to replenish the balloon during its descent. But the gaseous mass occupying a volume $V'-V$ at plane p' would occupy, on the ground, where the pressure is p, a volume of

$$(V'-V)\ p'/p.$$

If air be injected instead of gas, it is evident that the same volume will complete the filling; that is, the ballonets should contain on reaching the ground the following volume of air:

$$v=(V'-V)\ p'/p.$$

This should at least be the capacity of the ballonets.

Now
$$\frac{V'}{p}=\frac{V}{p'}=\frac{V'-V}{p-p'}$$

whence
$$\frac{V'-V}{V}=\frac{p-p'}{p'}.$$

Let the two factors be multiplied by p'/p, when the result is

$$\frac{v}{V}=\frac{p-p'}{p}. \qquad (2)$$

Let us return to formula (1) first established for small changes. The same formula therefore holds good for any changes whatever, if $\triangle V$ does not actually signify the increase of volume, but the reduction of that fraction of the volume at ground pressure. Such arithmetical reduction corresponds to the actual volume which fills the ballonet.

The minimum capacity which the ballonet should have is thus obtained by estimating v in formula (2), expressing in the second factor the relative depression corresponding to the maximum height the balloon can attain. The relative depression is obtained by referring to Hergesell's table.

For an altitude of 4,000 meters, for instance, $p'/p=$ about 6/10.

Therefore
$$\frac{p-p'}{p}=\frac{4}{10}$$

therefore
$$\frac{v}{V}=\frac{4}{10}.$$

The balloons which made land expeditions, sometimes attaining altitudes higher than 4,000 meters, had ballonets of a volume equal to half the total volume of the balloon. In all balloons the volume of the ballonets should be amply calculated.

Let us recall the fundamental diagrammatical law of the vertical movements of balloons in order to establish a relation between the volume of the ballonets and the total quantity of ballast carried. Col. Renard's law is as follows: The relative unballasting is equal to the relative depression.

If L, therefore, be the total weight of ballast which may be dropped, the pressure p' on the upper plane attained will be such that

$$\frac{L}{V\varphi}=\frac{p-p'}{p}$$

therefore

$$\frac{L}{V\varphi}=\frac{v}{V}.$$

If the lifting power φ on the ground is about 1.1, it is therefore evident that

$$v=\frac{L}{1.1}=.9\ L.$$

The volume v of the ballonets in cubic meters should be practically equal to the weight of all the ballast that can be dropped in kilos. This ballast naturally includes the gasoline, lubricating substance, ammunition, and reserve water. It is even prudent to add to the figure thus obtained the maximum thrust that may be obtained by employing dynamic means. When attacked or pursued, the dirigible would thus be able to seek safety in the maximum height of its navigation altitude. It would be a wise precautionary measure, moreover, to provide the ballonets with rip panels, which would enable them to communicate with the balloon in case of need.

EFFECT OF EXCESSIVE INTERIOR PRESSURE ON THE EQUILIBRIUM OF THE FLEXIBLE BALLOON.

In considering the vertical movements of balloons no account has been taken of interior overpressure, which but slightly affects the lifting power of the balloon.

In flexible dirigible balloons such overpressure attains a far greater value. An average value may be considered as equal to 40 millimeters of water. Let us see how it affects the equilibrium of the balloon:

1. The balloon is full of gas: *a*. The specific gravity of the gas is not proportionate to the external pressure p, but to $p+\triangle p$, $\triangle p$ being the overpressure.

Suppose the average specific gravity of the gas to be 150 grams at the plane of 10,000.

$A(\triangle p)=40$ corresponds to an increase of specific gravity equal to

$$150\times 40/10{,}000=.6 \text{ gr.}$$

The cubic meter of gas is thus rendered so much heavier. Its plane of equilibrium is thus lowered by a height of $h = 8{,}000\ 1/\varphi$, 1 being the increase of weight, and φ the solid weight which the cubic meter of gas is capable of raising; that is, its lifting power is as follows:

$$h = 8{,}000 \frac{.6}{1{,}250 - 150} = 4.4\text{m.}$$

Let us suppose that the altitude increases, the external pressure becoming p kilos per square meter, and the increase of weight per cubic meter of gas being equal to the fraction $40/p$ of the specific gravity of the moment, which is equal to

$$b_o\ p/10{,}000;$$

therefore $\quad 1 = b_o\ p/10{,}000.\quad 40/p = b_o.\quad 40/10{,}000.$

It is thus constant and equal in the numerical example chosen to

$$b_o = 150 \text{ grams to } .6 \text{ gr.}$$

But at plane p, the same deficit in height does not correspond to such increase of weight. It is here:

$$h' = 8{,}000\ 1/\varphi;$$

being the lifting power at plane p, we get

$$\varphi' = p/10{,}000.$$

Let, for instance, $p = 9{,}000$, which corresponds to a relative depression of 1/10 and to an elevation in height of 840 meters.

$$\varphi' = 1{,}100 \times 9/10 = 990 \text{ grams.}$$

Therefore

$$h' = 30{,}000 \times .6/990 = 5 \text{ meters.}$$

The lowering of the zone of equilibrium is 5 meters instead of 4.4 m., that is, 6 cms. more.

A relative unballasting which makes the balloon climb 840 meters without interior overpressure, that is, a relative overpressure of 1/10, makes a balloon maintained at an interior pressure of 40 millimeters climb a height of 40 millimeters less.

This result clearly shows the illusion of taking into account the interior overpressure and the increase of weight resulting therefrom in determining altitudes.

The interior overpressure is more remarkable for its variations and for the variations resulting therefrom in the volume of the balloon. This has already been pointed out from the weight standpoint. When the volume of the balloon diminishes, the interior pressure also

diminishes the volume subjected to the Archimedean thrust. The action of the valve evidently increases the weight of the balloon. When the *Fleurus* ejects gas from 30 millimeters to 20, it increases in weight by about 15 kilos. The same increase in weight is produced when the pressure drops from 30 to 20 in consequence of the interior pressure alone, or by the diminution of overheating. If, on the contrary, variations of volume and consequently of interior pressure are produced by the variation of atmospheric pressure or by variations of the outer and inner temperatures taking place at the same time, the balloon gas behaves like a gaseous mass free to undergo the variations of volume imposed upon it by atmospheric conditions (the compression of some millimeters more or less of water affects its weight to an unimportant degree). The loss of balance is therefore constant. So long as the balloon undergoes slight variations of altitude which cause variations in the inner pressure without necessitating exhaustion of gas, the balloon behaves like a slack balloon. In a continuous climbing movement, the decrease in the loss of balance is discontinued. It is determined only by the exhaustion of gas.

NOTE.—It has been stated above that the action of the valve increases the weight of the dirigible balloon. This is true within the limits in which the variations of interior overpressure act upon the exterior form of the balloon. The table of the variations of volume of the *Fleurus* shows that with the same increase of inner overpressure, the stronger is such overpressure, the less does the volume increase (12 cubic meters by 1 to 2 millimeters, 4 cubic meters by 10 to 11, and 1 cubic meter by 24 to 25 millimeters). It may thus happen, if it is necessary to navigate for the time being under very high pressure, that the diminution of interior overpressure due to the action of the valve entails only an unimportant diminution of the exterior volume. In such case, any escape of gas which is not followed up by the intake of an equal volume of air into the ballonets would lighten the balloon.

2. The balloon contains air in its ballonets. That air comes into juxtaposition with the gaseous mass, which always follows the variations of volume imposed by atmospheric conditions, in approximate accordance with the compression recognized as unimportant. The balloon into which air is driven, or out of which air is pumped by means of the valve, therefore behaves like a slack balloon, its loss of balance being constant.

It must, however, be noted that the same compression of 40 millimeters of water produces a greater absolute increase of gravity in a cubic meter of air than in a cubic meter of gas. Such increase is $1{,}250 \times 40/10{,}000 = 5$ grammes instead of .6 grammes. When a balloon descends and is equilibrated from the commencement of the movement, its gravity systematically increases by 5 grammes with

every cubic meter of air. The *Fleurus*, descending from a height of about 1,200 meters and having 1,000 cubic meters of air in its ballonets, increased in gravity by 5 kilos. A balloon like the *Champagne*, descending after a war expedition with more than 6,000 cubic meters of air in its ballonets, systematically increased by 30 kilos. That modification in the factors of equilibrium is even very slight for such a balloon. In any case, consequently, the effect of the compression of air or gas in a balloon is unimportant, and a dirigible, equilibrated at its highest navigable altitude, may be practically considered to be equilibrated at any lower altitude.

Accidental losses of balance, which may occur in the course of any ascent, are far more important. The art of the maneuver lies in averting them at any moment. They are especially due to variations of temperature, and in particular to the overheating factor. The latter is, however, of less importance than in free balloons, because the dirigible constantly renews the zones of air with which it comes in contact by its own movement, like a gyrostatic thermometer. But its great volume prevents it from assuming the exact temperature of the zones of air through which it passes. The overheating term thus exists in an unknown and variable sense, and is complicated by the continual phenomena of compression or expansion accompanying all the variations of interior overpressure. Its influence alone would not be sufficient to call for continual attention on the part of the pilot.

NOTE.—In conclusion, we shall here state an empiric rule that is particularly applicable to night ascents, but which generally suffices and does away with the need for constant recourse to formulas:

For 10,000 cubic meters 900 grammes must be dropped in order to rise 1 meter and to drive 1 cubic meter of air into the ballonet when descending 1 meter.

INDEX.

	Page.
Principle of Archimedes	5
Enumeration of the physical properties of gases	5
Density	6
Law of Mariotte	7
Law of Gay-Lussac	8
The mixture of gases	11
Lifting power of a gas	16
The weighing of a balloon	25
Principle of Pascal	27
The interior pressure in balloons	27
The case of dirigibles	32
Auxiliary supply balloons	35
Barometric leveling	37
The vertical movements of balloons	48
The fully inflated balloon	49
The slack balloon	57
Dropping of ballast and action of the valve	61
The effect of overheating	61
Diagrammatical construction	62
Application to dirigible airships	64
The effects of interior overpressure	67

ADDITIONAL COPIES

OF THIS PUBLICATION MAY BE PROCURED FROM
THE SUPERINTENDENT OF DOCUMENTS
GOVERNMENT PRINTING OFFICE
WASHINGTON, D. C.
AT
10 CENTS PER COPY
SUBSCRIPTION PRICE, 20 CENTS PER YEAR

▽